Proving the Validity of an Argument

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Introduction

Proving Validity: Examples

The Notion of Proof Rules

Proving Validity: More Examples

Fitch

Reasoning about Identity

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Reminder

In Fitch format, an argument takes the form

P_1	
	premises
P _n	
\overline{Q}	conclusion

- Such an argument is valid if conclusion Q is true whenever premises P₁...P_n are.
- A valid argument if sound if the premises are true.

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Proving and Disproving Validity

- To show that an argument is not valid, use counterexamples: a world where the premises are true, but conclusion false.
- How to show that an argument is valid? (topic of Section 2.2-2.4)
 - naive approach: consider all worlds where premise is true, and show that also conclusion is true.

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feasible approach: construct a proof

A Valid Argument

RightOf(b,c) LeftOf(d,e) b = d LeftOf(c,e) why?

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A Valid Argument

RightOf(b,c) LeftOf(d,e) b = d LeftOf(c,e) why?

Informal reasoning: (p.52)

We are told that *b* is to the right of *c*. So *c* must be to the left of *b*, since *right* of and *left* of are inverses of one another. And since b = d, *c* is left of *d*, by the indiscernibility of identicals. But we are also told that *d* is left of *e*, and consequently *c* is to the left of *e*, by the transitivity of *left* of. This is our desired conclusion.

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A Formal Proof

We establish a series of intermediate results:

3.
$$b = d$$

4. LeftOf(c,b) from 1, since LeftOf and RightOf are inverses

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- 5. LeftOf(c,d) from 4 and 3, using identity elimination
- 6. LeftOf(c,e) from 5 and 2, since LeftOf is transitive

Identity Elimination (= Elim)

- If b is a cube and b equals c, then also c is a cube
- If John is happy and John is the father of Max, then the father of Max is happy

In general (cf. p. 56)

$$| \begin{array}{c} P(t) \\ \cdots \\ t = u \\ \cdots \\ P(u) \end{array} \rangle$$

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A Valid Arithmetic Argument



A Valid Arithmetic Argument

1. <i>x</i> > 2	
2. 2 > 0	fact
3. <i>x</i> > 0	from 1 and 2, since $>$ is transitive
4. $x \cdot x > 2 \cdot x$	from 1 and 3, using law of arithmetic:
	multiplying by positive number preserves inequalities
5. $2 \cdot x = x + x$	fact
6. $x \cdot x > x + x$	from 4 and 5, using = Elim
	1. $x > 2$ 2. $2 > 0$ 3. $x > 0$ 4. $x \cdot x > 2 \cdot x$ 5. $2 \cdot x = x + x$ 6. $x \cdot x > x + x$

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Proof in Fitch

Converting previous proof into Fitch (software package):

- 1. RightOf(b,c)
- LeftOf(d,e)
- 3. b = d
- 4. LeftOf(c,b) from 1, since LeftOf and RightOf are inverses

- 5. LeftOf(c,d) from 4 and 3, using identity elimination
- 6. LeftOf(c,e) from 5 and 2, since LeftOf is transitive

Proof in Fitch

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 RightOf(b,c)
 LeftOf(d,e)
 b = d
 LeftOf(c,b)
 LeftOf(c,d)
 LeftOf(c,c)
 from 4 and 3, using identity elimination from 5 and 2, since LeftOf is transitive

Proof in Fitch

Converting previous proof into Fitch (software package):

- 1. RightOf(b,c)
- LeftOf(d,e)
- 3. b = d
- 4. LeftOf(c,b) Ana Con: 1
- 5. LeftOf(c,d) = Elim: 4, 3
- 6. LeftOf(c,e) from 5 and 2, since LeftOf is transitive

Proof in Fitch

Converting previous proof into Fitch (software package):

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- 2. LeftOf(d,e)
- 3. b = d
- 4. LeftOf(c,b) Ana Con: 1
- 5. LeftOf(c,d) = Elim: 4, 3
- 6. LeftOf(c,e) Ana Con: 5, 2

Limitations on Fitch

- Fitch is tuned for Tarski's World, with Ana Con modeling the laws of the block world
- Fitch is not tuned for arithmetic.
 Thus Fitch cannot handle the proof that x · x > x + x

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- Ana Con is very strong, so do not use it for exercises, unless explicitly allowed (neither use Taut Con or FO Con) The Grade Grinder will report unauthorized use of Ana Con!
- System F is not tuned for Tarski's world, and therefore has no special proof rules for the predicates there, like LeftOf
- But since the identity relation "=" is used in all domains of discourse, system F has special rules for that predicate

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System F, vs. Fitch

	system F (p.54)	Fitch (p.58)
what is?	formal system	software package
application domain	general purpose	tuned for Tarski's world
Proof rules:		
general	= Elim, = Intro	= Elim, = Intro
-		
	\wedge Elim, \wedge Intro	\wedge Elim, \wedge Intro
-	\land Elim, \land Intro etc.	∧ Elim, ∧ Intro etc.
shortcuts	∧ Elim, ∧ Intro etc. can be added	∧ Elim, ∧ Intro etc. Taut Con, FO Con
shortcuts specific	 ∧ Elim, ∧ Intro etc. can be added can be added 	∧ Elim, ∧ Intro etc. Taut Con, FO Con Ana Con

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Identity Introduction (= Intro)

Rule described p.55 is amazingly simple:

 $\triangleright \mid t = t$

This says that the identity relation is reflexive.

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Properties of Identity

Symmetry

$$a = b$$

 $b = a$

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Properties of Identity

Symmetry

$$a = b$$
$$b = a$$

Transitivity

$$a = b$$
$$b = c$$
$$a = c$$

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Properties of Identity

Symmetry

Transitivity

$$a = b$$
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Properties of Identity

Symmetry

Transitivity

1.
$$a = b$$

2. $b = c$
3. $a = c$ = Elim: 1, 2