The Validity and Soundness of Arguments

Torben Amtoft
Kansas State University

Road Map

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1. help you learn language of first-order logic (FOL)
2. help you learn notion of logical consequence

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- Chapter 1 takes the first step of (1)
- Chapter 2 takes the first step of (2)

Atomic Sentences

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	functions	predicates
result is	object	truth value
spelling is	lower case	capitalized
can be nested?	yes	no

Example Worlds

	constants	functions		predicates	
		arity 1	arity 2	arity 1	arity 2
Arithmetic	0,1,2,...	sin, cos	+,-		$<$
Family	`max`, `claire`	`father`		`Pet`	`Older`
Tarski's World	`a`, `b`, ...			`Cube`	`LeftOf`

- many functions and predicates with arity 2 are written infix: $x + y$, $x < y$, $x = y$
- functions can be added to Tarski's world (p.33, and homework exercises 1.13 & 1.14)
- The identity predicate "=" is relevant in all worlds!

Motivation

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> *one person presenting a series of statements in which one, the conclusion, is meant to be a consequence of the others, called the premises.*

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Fitch format

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---
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Valid and Sound Arguments

Socrates is a man
All men are mortal

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Valid and Sound Arguments

Socrates is a man
All men are mortal
∴ Socrates is mortal

This classical argument is

- valid: it is not possible for the conclusion to be false if the premises are true.

Valid and Sound Arguments

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All men are mortal	true (biology)
Socrates is mortal	

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- sound: it is valid, and its premises are true.

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Socrates is mortal	true (history: hemlock, 399 BC)

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- sound: it is valid, and its premises are true.
(so also its conclusion is true)

Unsound Arguments

Scruffy is a man
All men are mortal

Scruffy is mortal

This argument is

- valid, as same structure as the previous argument

Unsound Arguments

Scruffy is a man	false
All men are mortal	true
Scruffy is mortal	true

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  (conclusion happens to be true)

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Red Sox win the World Series each year	false
Red Sox will win the 2004 World Series	?

This is also valid, but, alas, not sound.

Invalid Arguments

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All men are mortal

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This argument has a different structure than what we have seen, and is invalid.

Invalid Arguments

Socrates is mortal	true
All men are mortal	true
Socrates is a man	false

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Counterexample: Socrates might be a dog

Invalid Arguments

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All men are mortal

Socrates is a man

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Counterexample: Socrates might be a dog
To decide whether an argument is

- valid: it is sufficient to examine the structure of the argument
- sound: we must examine history, biology, baseball, etc.

Therefore the focus of logic, and this course, is on validity of argument, rather than on soundness.

Counterexamples (Section 2.5)

Given a purported argument, a counterexample is

- a world where the premises are true but the conclusion is false
- enough to show that the argument is invalid: the conclusion does *not* follow from the premises (is non sequitur).

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Submitting Counterexamples

In homeworks, you'll often be given an argument and asked to submit a world that serves as a counterexample.

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LeftOf(a,b)
SameSize(b,c)
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LeftOf(a,c)
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Counterexample: none, as argument is valid