

A Theorem Proving Approach to Analysis of Secure Information Flow

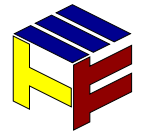
A Paper by *Ádám Darvas, Rainer Hänle, and David
Sands*

Georg Jung

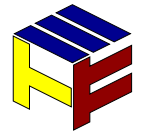
`jung@cis.ksu.edu`

Department of Computing and Information Sciences, Kansas State University

Overview

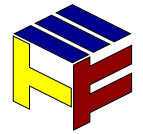


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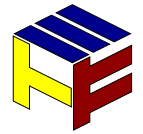
- Introduction to Dynamic Logic

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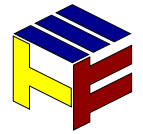
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- The KeY Theorem Prover

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- Application of the Framework

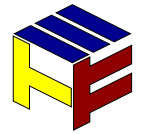
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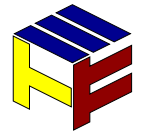


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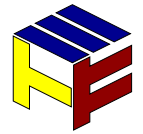
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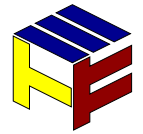
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- The theorem prover running “life”
- A meaningful example



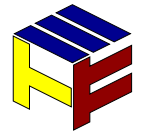
Structure of the Dynamic Logic

Basics of Dynamic Logic



Dynamic Logic (**DL**) (David Harel, Dexter Kozen, Jerzy Tiuryn, 1984) was designed to reason about programs.

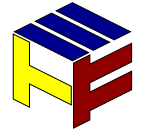
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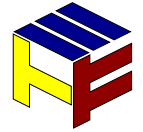


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DL combines

- First Order Predicate Logic

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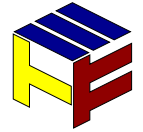


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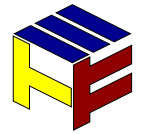


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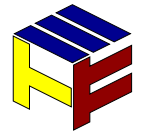
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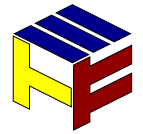
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DL instead features syntactic constructs to explicitly change a valuation. These constructs are referred to as *programs*.

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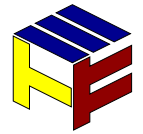
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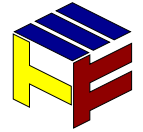
In this presentation I describe a special **DL** called **JAVA CARD DL**, which is introduced by the referred paper.

Modalities of JAVA CARD DL



DL offers two modalities for every program p :

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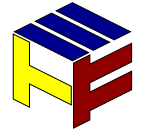


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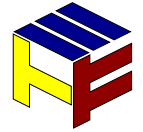
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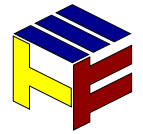


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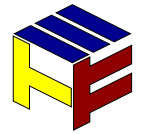
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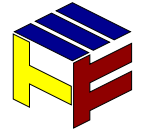
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- For technical reasons there is also the *update* $\{loc := val\}$ which has the same semantics as $\langle loc = val; \rangle$, only that the evaluation of val cannot have side effects.

Variable Types in DL

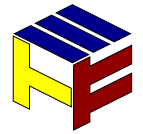


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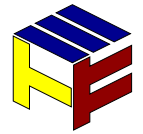
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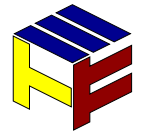
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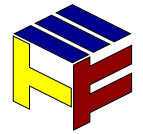
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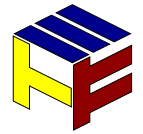


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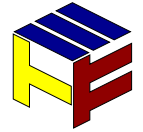
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To express the quantification $\forall \mathbf{x}. \langle \mathbf{p}(\mathbf{x}) \rangle \psi(\mathbf{x})$, we hence have to write

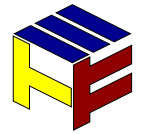
$$\langle \text{int } \mathbf{x}; \rangle (\forall l : \text{int}. \{ \mathbf{x} := l \} \langle \mathbf{p}(\mathbf{x}) \rangle \psi(l, \mathbf{x}))$$

Comparison to Hoare Logic



The **DL** formula $\phi \rightarrow \langle p \rangle \psi$ is similar to the *total correctness* Hoare triple $\{\phi\}p\{\psi\}$ (C. A. R. Hoare. An axiomatic basis for computer programming. ACM 1969)

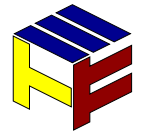
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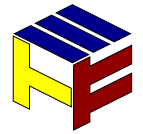


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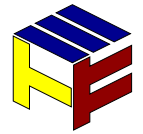
Hoare Logic does not provide for characterization of termination behavior.

Example 1



Consider two variables h and l . We want to express that no information flows from h to l :

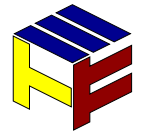
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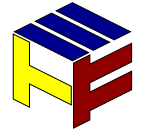
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Remember that the update has pure technical reasons. For the presentation we can write:

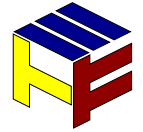
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Example 2



In addition to the independence of the result value r from any validation of the variable h we want to express that no information about h leaks from the termination behavior of $\langle p \rangle$.

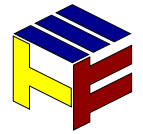
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$$\forall l. \exists r. \forall h. \langle p \rangle r = 1$$

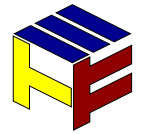
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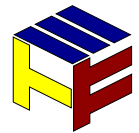
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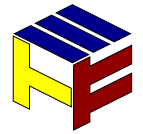
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Note that this formula contains an implicit use of the $[\cdot]$ modality and can therefore not be handled by the present implementation of KeY.



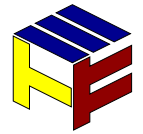
The *KeY* Theorem Prover

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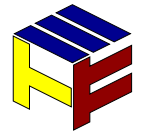
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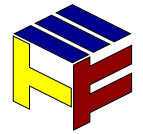
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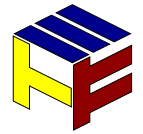
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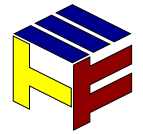


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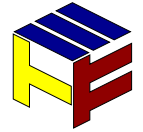


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- Existentially quantified variables (which become universally quantified through implicit negation) are replaced by *meta variables*, so that they can be instantiated later (*delayed instantiation*).



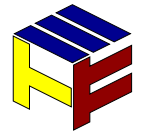
Application of the Framework

A Toy Example



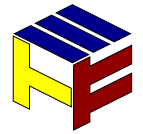
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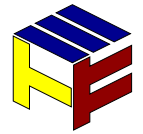
To prove the formula $\forall l. \exists r. \forall h. \langle p \rangle r = l$ it has to be negated:

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The paper claims that the program “ $l = h;$ ” can be proven insecure in 14 steps without user interaction. I was not yet able to reproduce the proof and the paper does not give any further information but the goal which fails:

$$\exists r. \forall h. r = h$$

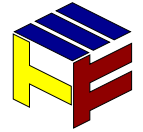
More Realistic



The following example is considered in the paper:

```
class Account {  
    private int balance;  
    public boolean extraService;  
  
    private void writeBalance (int amount) {  
        if (amount >= 10000) extraService = true;  
        else extraService = false;  
        balance = amount; }  
    ...  
}
```

More Realistic



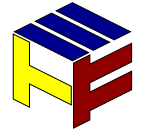
...

```
private int readBalance () {  
    return balance; }
```

```
public boolean readExtra () {  
    return extraService; }
```

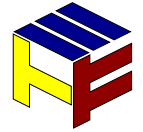
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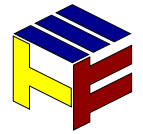
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We formalize the goal:

```
⟨Account o = new Account (); int amount; boolean result;⟩
```

More Realistic

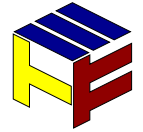


We formalize the goal:

```
⟨Account o = new Account (); int amount; boolean result;⟩
```

$\forall e : \text{boolean} . \exists r : \text{boolean} . \forall a : \text{int} .$

More Realistic



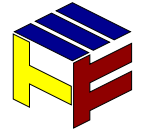
We formalize the goal:

```
⟨Account o = new Account (); int amount; boolean result;⟩
```

$\forall e : \text{boolean}. \exists r : \text{boolean}. \forall a : \text{int}.$

$\{o.\text{extraService} := e\} \{ \text{amount} := a \}$

More Realistic



We formalize the goal:

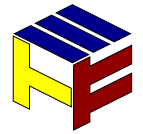
```
⟨Account o = new Account (); int amount; boolean result;⟩
```

$\forall e : \text{boolean} . \exists r : \text{boolean} . \forall a : \text{int} .$

$\{o.\text{extraService} := e\} \{ \text{amount} := a \}$

$\langle o.\text{writeBalance}(\text{amount}); \text{result} = o.\text{readExtra}(); \rangle$

More Realistic



We formalize the goal:

```
⟨Account o = new Account (); int amount; boolean result;⟩
```

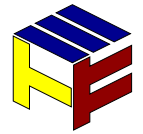
$\forall e : \text{boolean}. \exists r : \text{boolean}. \forall a : \text{int}.$

$\{o.\text{extraService} := e\} \{ \text{amount} := a \}$

$\langle o.\text{writeBalance}(\text{amount}); \text{result} = o.\text{readExtra}(); \rangle$

$r = \text{result}$

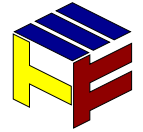
Exceptions



The following example is considered in the paper:

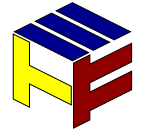
```
l = true;  
try {  
    if (h) throw new Exception ();  
    l = false;  
}  
catch (Exception e) {}
```

Exceptions



Again, we formalize the goal:

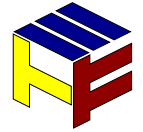
Exceptions



Again, we formalize the goal:

```
⟨boolean l; boolean h;⟩
```

Exceptions

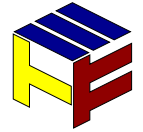


Again, we formalize the goal:

$\langle \text{boolean } l; \text{ boolean } h; \rangle$

$\forall x : \text{boolean}. \exists r : \text{boolean}. \forall a : \text{boolean}.$

Exceptions



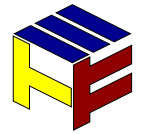
Again, we formalize the goal:

$\langle \text{boolean } l; \text{ boolean } h; \rangle$

$\forall x : \text{boolean}. \exists r : \text{boolean}. \forall a : \text{boolean}.$

$\{ l := x \} \{ h := y \}$

Exceptions



Again, we formalize the goal:

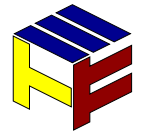
$\langle \text{boolean } l; \text{ boolean } h; \rangle$

$\forall x : \text{boolean}. \exists r : \text{boolean}. \forall a : \text{boolean}.$

$\{ l := x \} \{ h := y \}$

$\langle p \rangle$

Exceptions



Again, we formalize the goal:

$\langle \text{boolean } l; \text{ boolean } h; \rangle$

$\forall x : \text{boolean}. \exists r : \text{boolean}. \forall a : \text{boolean}.$

$\{ l := x \} \{ h := y \}$

$\langle p \rangle$

$r = l$