**Chapter 3**

# Arithmetic and Variables



*Traditional computer programs* calculate*: They calculate monthly payments on a loan, compute the roots of a quadratic equation, or convert dollars into euros. Such calculations rely on two concepts from algebra—arithmetic operations and variables—and this chapter develops both as they appear in programming.*

*The chapter's objectives are to*

- *Review the principles of arithmetic computation and introduce the notion of* data type*, which ensures consistency within computations.*
- *Introduce* computer variables *and their traditional uses.*
- *Provide a simplistic but effective means to supply* input data *to a program, so that one program can be used over and over to perform a family of related calculations.*

## 3.1 Integer Arithmetic

An electronic computer is an "overgrown" pocket calculator, so it is no surprise that we can write computer programs that tell a computer to calculate on numbers. We will learn to write such programs in Java by studying a classic example: calculating the value of the coins in your pocket.

How do you calculate the value of your spare change? No doubt, you separate into groups your quarter coins (25-cent pieces), dimes (10-cent pieces), nickels (5-cent pieces), and pennies (1-cent pieces). Once you count the quantity of each, you multiple each quantity by the value of the coin and total the amounts.

For example, if you have 9 quarters, 2 dimes, no nickels, and 6 pennies, you would calculate this total:

```
(9 times 25) plus (2 times 10) plus (0 times 5) plus (6 times 1)
```

which totals to 251 cents, that is, $2.51.

If your arithmetic skills are weak, you can embed the above computation into a simple Java program and have the computer do it for you, because the Java language lets you write arithmetic expressions like `9 * 25` (the `*` means multiplication) and `(9 * 25) + (2 * 10)` (the `+` means addition), and so on—symbols like `*` and `+` are called *operators*, and the operators' arguments (e.g., `9` and `25` in `9 * 25`) are their *operands*.

Here is the program that does the calculation:

```
/** Total computes the amount of change I have, based on
  * 9 quarters, 2 dimes, no nickels, and 6 pennies */
public class Total
{ public static void main(String[] args)
  { System.out.println("For 9 quarters, 2 dimes, no nickels, and 6 pennies,");
    System.out.print("the total is ");
    System.out.println( (9 * 25) + (2 * 10) + (0 * 5) + (6 * 1) );
  }
}
```

This program prints in the command window:

```
For 9 quarters, 2 dimes, no nickels, and 6 pennies,
the total is 251
```

Of course, the crucial statement is

```
System.out.println( (9 * 25) + (2 * 10) + (0 * 5) + (6 * 1) );
```

which embeds the arithmetic calculation as the argument to `System.out`'s `println` method. As we discovered in the previous chapter, *the argument part of a message is always calculated to its answer before the message is sent.* In the present case, it

means that the multiplications and additions are computed to 251 before `System.out` is told to print.

If a comment, like the one in the above example, extends across multiple lines, we begin each new line with an asterisk; the reason is explained in Chapter 5.

In the Java language, whole numbers, like 6, 0, 251, and -3, are called *integers*. We see momentarily that the Java language uses the keyword, `int`, to designate the collection of integers.

*Begin Footnote:* More precisely, `int` is Java's name for those integers that can be binary coded within one computer word, that is, the integers in the range of -2 billion to 2 billion. *End Footnote.*

Of course, you can write Java programs to do other numerical calculations. For the moment, consider addition (`+`), subtraction (`-`), and multiplication (`*`); you can use these in a Java program the same way you write them on paper, e.g.,

```
1 + ((2 - 4) * 3) + 5
```

is a Java arithmetic expression that calculates to 0. You can test this example simply enough:

```
public class Test
{ public static void main(String[] args)
  { System.out.println(1 + ((2 - 4) * 3) + 5); }
}
```

When the computer executes this program, it calculates the expression from left to right, respecting the parentheses; if we could peer inside the computer's processor, we might see these steps:

```
    1 + ((2 - 4) * 3) + 5
=>  1 + ((-2) * 3) + 5
=>  1 + (-6) + 5
=>  -5 + 5  =>  0
```

*When you write arithmetic expressions, insert parentheses to make clear the order in which operations should occur.*

### Exercises

Calculate each of these expressions as the computer would do; show all the calculation steps. After you have finished the calculations, write each of them within a Java application, like the one in this section.

1. `6 * ((-2 + 3) * (2 - 1))`
2. `6 * (-2 + 3) * (2 - 1)`
3. `6 * -2 + 3 * (2 - 1)`
4. `6 * -2 + 3 * 2 - 1`

## 3.2 Named Quantities: Variables

You will not enjoy using the previous change-counting program, `Total`, over and over, because it will be difficult to alter it each time you come home with a new set of coins in your pocket. We can improve the situation by giving names to the quantities of quarters, dimes, nickels, and pennies. These names are called *computer variables*, or *variables* for short.

Used in the simplest way, a variable is a just a name for an integer, just like "Lassie" is just a name for your pet dog.

*Begin Footnote:* But as already noted in Chapter 2, a variable name is in reality a storage cell that holds the named integer. For the moment, don't worry about the storage cell; this detail is revealed only when necessary in a later section. *End Footnote*

To give the name, `quarters`, to the integer, 9, we write this Java statement:

```
int quarters = 9;
```

The keyword, `int`, signifies that `quarters` names an integer; indeed, it names 9.

This form of statement is called a *variable declaration*; more precisely, it is a *variable initialization*, because it creates a new variable and initializes it with a numerical value. The keyword, `int`, is a *data type*—it names a "type" or "species" of data value, namely the integers. The keyword, `int`, tells the Java compiler that the value of `quarters`, whatever it might be, *must* be from the data type `int`.

Since the variable, `quarters`, names an integer, we use it just like an integer, e.g.,

```
System.out.println(quarters * 25);
```

would calculate and print 225. When we use a variable like `quarters`, in an expression, we say we are *referencing* it.

Indeed, when the Java compiler examines a reference to a variable, like the one in the previous statement, it verifies that the variable's data type (here, `int`) is acceptable to the context where the variable is referenced (here, the variable is multiplied by `25`—this is an acceptable context for referencing variable `quarters`). This examination by the Java compiler, called *data-type checking*—prevents problems which might arise, say, if an `int`-typed variable was referenced in a context where a string was expected.

When we need to distinguish between variables, like `quarters`, that denote integers, and actual integers like `6` and `225`, we call the latter *literals*.

The name you choose for a variable is called an *identifier*; it can be any sequence of letters, numerals, or the underscore, `_`, or even a dollar sign, `$` (not recommended!), as long the name does not begin with a numeral, e.g., `quarters` or `QUArTers` or `my_4_quarters`. But Java keywords, like `public` and `class`, cannot be used as variable names.

Figure 3.1: change computing program

```
/** TotalVariables  computes the amount of change I have, based on the values
  * named by the four variables,  quarters, dimes, nickels, and  pennies. */
public class TotalVariables
{ public static void main(String[] args)
  { int quarters = 9;
    int dimes = 2;
    int nickels = 0;
    int pennies = 6;

    System.out.println("For these quantities of coins:");
    System.out.print("Quarters = ");
    System.out.println(quarters);
    System.out.print("Dimes = ");
    System.out.println(dimes);
    System.out.print("Nickels = ");
    System.out.println(nickels);
    System.out.print("Pennies = ");
    System.out.println(pennies);
    System.out.print("The total is ");
    System.out.println( (quarters * 25) + (dimes * 10)
                          + (nickels * 5) + (pennies * 1) );
  }
}
```

Here is an improved version of the change-counting example in the previous section; it names each of the four quantities and prints a detailed description of the quantities and their total. Here is what the program prints:

```
For these quantities of coins:
Quarters = 9
Dimes = 2
Nickels = 0
Pennies = 6
The total is 251
```

The program itself appears in Figure 1. Note that long statements, like the last one in the Figure, can extend to multiple text lines, because the semicolon marks a statement's end.

It is easy to see the four variables and understand how they are referenced for change calculation. Now, if you wish to modify the program and calculate a total for different quantities of coins, *all you must do is change the appropriate variable initializations*, and the remainder of the program works correctly.

We finish the change calculation example with this last improvement: We make the total print as dollars and cents, like so:

```
For these quantities of coins:
Quarters = 9
Dimes = 2
Nickels = 0
Pennies = 6
The total is 2 dollars and 51 cents
```

The secret to converting a cents amount to dollars-and-cents is: divide the cents amount by 100, calculating how many *whole* dollars can be extracted—this is the *quotient* of the result. In the Java language, an integer quotient is computed by the integer-division operator, `/`. For example,

`251 / 100` computes to `2`,

because 100 can be extracted from 251 at most 2 times (and leaving a nonnegative remainder).

The remainder part of an integer divison is calculated by the remainder ("modulo") operator, `%`:

`251 % 100` computes to `51`,

because after groups of 100 are extracted from 251 as many as possible, 51 remains as the leftover.

To use this technique, we write these statements:

```
int total = (quarters * 25) + (dimes * 10) + (nickels * 5) + (pennies * 1);
System.out.print("The total is ");
System.out.print(total / 100);
System.out.print(" dollars and ");
System.out.print(total % 100);
System.out.println(" cents");
```

The variable initialization lets `total` name the total of the change, and the statements that follow apply the quotient and remainder operations to `total`. These statements print

```
The total is 2 dollars and 51 cents
```

The long sequence of `print` statements just seen can be compressed into just one with this Java "trick": When a textual string is "added" to another string or an integer, a longer string is formed. For example

```
System.out.println("Hello" + "!");
System.out.println("Hello" + (48 + 1));
```

Figure 3.2: Calculating the value of change

```
/** Total computes the amount of change I have, based on
    the values named by the variables,
    quarters, dimes,  nickels,  and   pennies */
public class Total
{ public static void main(String[] args)
  { int quarters = 5;
    int dimes = 2;
    int nickels = 0;
    int pennies = 6;

    System.out.println("For these quantities of coins:");
    System.out.println("Quarters = " + quarters);
    System.out.println("Dimes = " + dimes);
    System.out.println("Nickels = " + nickels);
    System.out.println("Pennies = " + pennies);
    int total = (quarters * 25) + (dimes * 10) + (nickels * 5) + (pennies * 1);
    System.out.println("The total is " + (total / 100) + " dollars and "
       + (total % 100) + " cents");
  }
}
```

will print the lines

```
Hello!
Hello49
```

In Java, the + operator is *overloaded* to represent both numerical addition as well as textual string concatenation (appending one string to another). Perhaps this is not a wise use of the + symbol, but it is certainly convenient!

The above trick lets us print the dollars-cents total as one long statement:

```
System.out.println("The total is " + (total / 100) + " dollars and "
   + (total % 100) + " cents");
```

Figure 2 exploits everything we have learned in the final version of the change-calculation program.

## Exercises

1. Modify the application in Figure 1 so that it calculates the value of 3 quarters and 12 nickels; compile it and execute it.

2. Modify the application in Figure 1 so that it displays the value of each group of coins and then the value of all the coins. Try the application for 4 quarters, 1 nickel, and 1 penny. The application should reply:

```
For these quantities of coins:
Quarters = 4, worth 100 cents
Dimes = 0, worth 0 cents
Nickels = 1, worth 5 cents
Pennies = 1, worth 1 cents
The total is 106 cents
```

3. Modify Figure 2 so that it displays its output in decimal format, e.g.,

```
The total is $2.51
```

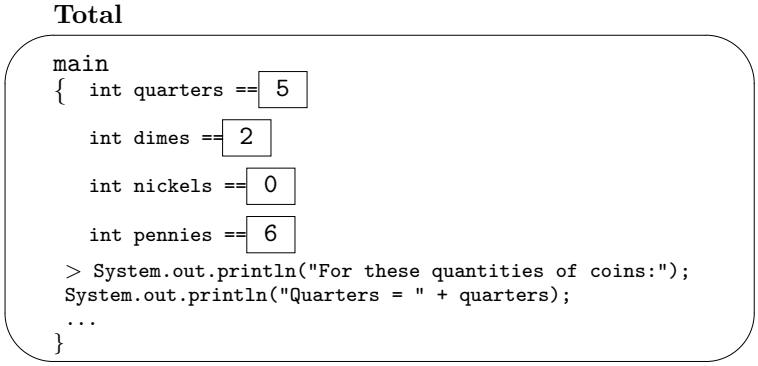
   Explain what happens when the modified application is used with the quantities of coins listed in the previous Exercise. (We will repair the difficulty in the section, "Converting between Strings and Numbers and Formatting.")

4. For practice, write an application in Java that prints a multiplication table of all pairs of numbers between 1 and 3, that is, 1*1, 1*2, and so on, up to 3*3.

### 3.2.1 Variables Can Vary: Assignments

As the examples in the previous section showed, variables name numerical quantities. But they can do more than that—*the value that a variable names can change as a computation proceeds*. Unlike the variables in algebra, computer variables can truly vary.

Here's why: When a variable is initialized, a *storage cell* in primary storage is created, and the number named by the variable is placed into the cell. Here is a picture of the object created in primary storage by `class Total` in Figure 2 just after the four variables are initialized:



The diagram shows that each number is saved in a cell. When a statement like `System.out.println("Quarters = " + quarters)` is executed, the reference to `quarters` causes the computer to look inside the cell named by `quarters` and use the integer therein.

The above picture is important, because the Java language allows you to change the value held in a variable's cell; you use a statement called an *assignment*. Here is a small example, where a variable, `money`, is initialized to 100, and then the money is completely withdrawn:

```
int money = 100;
System.out.println(money);
money = 0;
System.out.println(money);
```

The statement, `money = 0`, is the assignment; it alters the value within the variable cell that already exists. This sequence of statements displays in the command window,

```
100
0
```

because the cell's initial value, 100, was overwritten by 0 by the assignment.

A more interesting example creates a variable to hold money and then deposits 50 more into it:

```
int money = 100;
System.out.println(money);
money = money + 50;
System.out.println(money);
```

First, `100` is printed, and then the assignment, `money = money + 50` calculates a new value for `money`'s cell, namely, the previous value in the cell plus 50. Thus, `150` prints the second time the value in `money`'s cell is consulted.

We use the power of assignments to write another classic program with change: Say that we desire a program that prints the coins one needs to convert a dollars-and-cents amount into coins. For example, if we have 3 dollars and 46 cents, the program reports that we receive this change:

```
quarters = 13
dimes = 2
nickels = 0
pennies = 1
```

If you solved this problem on a street corner, you would first count the number of quarter-dollar coins that are needed and subtract the amount of quarters from the starting money (e.g., 3.46 - (13*0.25) = 0.21). Then, you would repeat the process for dimes (0.21 - (2*0.10) = 0.01), nickels (0.01 - (0*0.5) = 0.01) and pennies. The value of the remaining money decreases from 3.46 to 0.21 to 0.01 to 0.

Here is the algorithm for the change-making method:

1. Set the starting value of `money`.
2. Subtract the maximum number of quarters from `money`, and print the quantity of quarters extracted.
3. Subtract the maximum number of dimes from `money`, and print the quantity of dimes extracted.
4. Subtract the maximum number of nickels from `money`, and print the quantity of nickels extracted.
5. The remainder of `money` is printed as pennies.

Here is how we might write the first step in Java:

```
int dollars = 3;
int cents = 46;
int money = (dollars * 100) + cents;
```

The second step of the algorithm is cleverly written as follows:

```
System.out.println("quarters = " + (money / 25));
money = money % 25;
```

These statements exploit *integer division*, `/`, and *integer modulo*, `%`.

In the first statement, `money / 25` calculates the maximum number of quarters to extract from `money`. There are two important points:

1. The integer division operator, `/`, calculates *integer quotient*—the number of times 25 can be subtracted from `money` without leaving a negative remainder. In the example, since `money`'s cell holds 346, the quotient is 13, because a maximum of 13 quarters (13*25 = 325) can be wholly subtracted from 346 without leaving a negative remainder. Here are additional examples for intuition:
   - `14 / 3` computes to 4 (and the remainder, 2, is forgotten)
   - `6 / 3` computes to 2 (and there is no remainder)
   - `4 / 5` computes to 0, because 5 cannot be wholly subtracted from 4
2. The calculation of the division, `money / 25`, does *not* alter the value in `money`'s cell, which remains 346—only an assignment statement can change the value in `money`'s cell.

The second statement, `money = money % 25`, deals with the second point just mentioned: Since we have calculated and printed that 13 whole quarters can be extracted from the amount of money, we must reset the value in `money`'s cell to the remainder. This can be done either of two ways:

Figure 3.3: change-making program

```
/**  MakeChange calculates the change for the amounts in variables
  *   dollars  and  cents.  */
public class MakeChange
{ public static void main(String[] args)
  { int dollars = 3;
    int cents = 46;
    int money = (dollars * 100) + cents;
    System.out.println("quarters = " + (money / 25));
    money = money % 25;
    System.out.println("dimes = " + (money / 10));
    money = money % 10;
    System.out.println("nickels = " + (money / 5));
    money = money % 5;
    System.out.println("pennies = " + money); }
}
```

1. By the statement,

   ```
   money = money - ((money / 25) * 25);
   ```

   which calculates the monetary value of the extracted quarters and subtracts this from `money`.

2. By the more elegant statement,

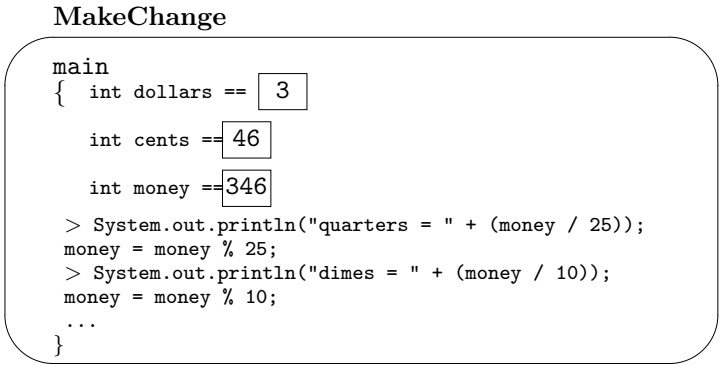
   ```
   money = money % 25;
   ```

   whose modulo operator, `%`, calculates the *integer remainder* of dividing `money` by `25` and assigns it to `money`. In this example, the remainder from performing 346/25 is of course 21. Here are additional examples of computing remainders:

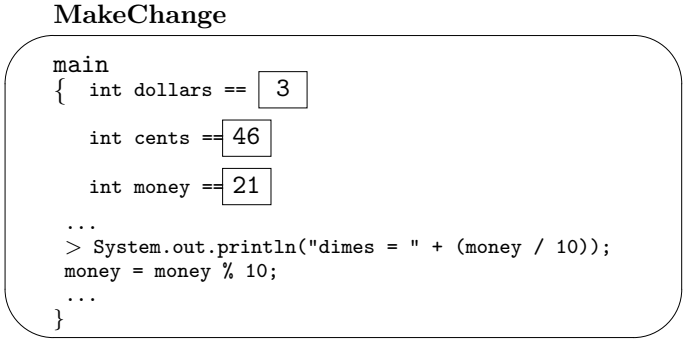
   - `14 % 3` computes to 2
   - `6 % 3` computes to 0
   - `4 % 5` computes to 4

The combination of the integer quotient and remainder operations calculates the correct quantity of quarters. We apply this same technique for computing dimes and nickels and see the resulting program in Figure 3.

When the application starts, the `main` method executes, and the three variable initializations create three cells:




At this point, `quarters = 13` is printed, because `346 / 25` gives the quotient, 13. The assignment, `money = money % 25`, that follows causes the value in `money`'s cell to decrease, because `money % 25` computes to 21:




The value in `money`'s cell changes twice more, as dimes and nickels are extracted from it.

The circular-appearing assignments in the example, like `money = money % 25`, might distress you a bit, because they look like algebraic equations, but they are *not*. The semantics of the previous assignment statement proceeds in these three steps:

1. The variable cell named by `money` is located
2. The expression, `money % 25`, is computed to an answer, referencing the value that currently resides in `money`'s cell.
3. The answer from the previous step is placed into `money's` cell, *destroying whatever value formerly resided there.*

It is unfortunate that the equals sign is used to write an assignment statement—in Java, `=` does not mean "equals"!

Finally, because of the existence of the assignment statement, it is possible to declare a valueless variable in one statement and assign to it in another:

```
int dollars;  // this is a declaration of  dollars;  it creates a cell
dollars = 3;  // this assigns 3 to the cell
```

but if possible, declare and initialize a variable in one and the same statement.

**Exercises**

1. Revise the program, in Figure 3 so that it makes change in terms of five-dollar bills, one-dollar bills, quarters, dimes, nickels, and pennies.

2. What are the results of these expressions? `6/4`; `6%4`; `7/4`; `7%4`; `8/4`; `8%4`; `6/-4`; `-6%4`; `6%-4`.

3. Use algebra to explain why the assignment, `money = money % 25`, in the `MakeChange` program correctly deducts the number of a quarters from the starting amount of `money`.

4. Write this sequence of statements in Java:

   - A variable, `my_money`, is initialized to 12.
   - `my_money` is reduced by 5.
   - `my_money` is doubled.
   - `my_money` is reset to 1.
   - The value of `my_money` is sent in a `println` message to `System.out`.

5. Write an execution trace of this application:

```
public class Exercise3
{ public static void main(String[] args)
  { int x = 12;
    int y = x + 1;
    x = x + y;
    y = x;
    System.out.println(x + " equals " + y);
  }
}
```

## 3.3 Arithmetic with Fractions: Doubles

Here is the formula for converting degrees Celsius into degrees Fahrenheit:

```
f = (9.0/5.0)*c + 32
```

Once we decide on a value for `c`, the degrees Celsius, we can calculate the value of `f`, the degrees in Fahrenheit. So, if `c` is 22, we calculate that `f` is 71.6, a fractional number. In Java, fractional numbers are called *doubles* ("double-precision" fractional numbers). The Java keyword for the data type of doubles is `double`.

Double literals are conveniently written as decimal fractions, e.g., `9.0` and `-3.14159`, or as mantissa-exponent pairs, e.g., `0.0314159E+2`, which denotes 3.14159 (that is, 0.0314159 times $10^2$) and `3E-6`, which denotes .000003 (that is, 3 times $1/10^6$).

Java allows the classic arithmetic operations on doubles: addition (whose operator is `+`), subtraction (`-`), multiplication (`*`), and division (`/`). Given fractional (double) operands, the operations calculate doubles as results. For example, we can write `(1.2 + (-2.1 / 8.4)) * 0.33`, and the computer calculates

```
    (1.2 + (-2.1 / 8.4)) * 0.33
=>  (1.2 + (-0.25)) * 0.33
=>  (0.95) * 0.33  =>  0.3135
```

Note that division, `/`, on fractional numbers produces fractional results—for example, `7.2 / 3.2` computes to `2.25`, and `6.0 / 4.0` computes to `1.5`.

Returning to the temperature conversion formula, we write it as a Java application. The key statements are these two:

```
int c = 22;
double f = ((9.0/5.0) * c) + 32;
```

The variable initialization for `f` starts with the keyword, `double`, to show that `f` names a double.

It is acceptable to mix integers, like `c`, with doubles in an expression—the result will be a double, here, 71.6. (This is the case even when the fraction part of the result is zero: `2.5 * 4` computes to `10.0`.)

The preceding examples present an important point about the basic arithmetic operations, `+`, `-`, `*`, and `/`:

- When an arithmetic operation, like `+`, is applied to two arguments of data type `int`, e.g., `3 + 2`, then the result of the operation is guaranteed to be an integer—have data type `int`—as well.

- When an arithmetic operation is applied to two numerical arguments, and at least one of the arguments has type `double`, e.g., `3 + 2.5`, then the result is guaranteed to have type `double`.

The "guarantees" mentioned above are used by the Java compiler to track the data types of the results of expressions without actually calculating the expressions themselves. We examine this issue in a moment.

The completed temperature program is simple; see Figure 4. When started, this application displays,

Figure 3.4: temperature conversion

```
/**  CelsiusToFahrenheit converts a Celsius value to Fahrenheit.  */
public class CelsiusToFahrenheit
{ public static void main(String[] args)
  { int c = 22;   // the degrees Celsisus
    double f = ((9.0/5.0) * c) + 32;
    System.out.println("For Celsius degrees " + c + ",");
    System.out.println("Degrees Fahrenheit = " + f);
  }
}
```

```
For Celsius degrees 22,
Degrees Fahrenheit = 71.6
```

In addition to fractional representation, the Java language lets you write doubles in *exponential notation*, where very large and very small doubles are written in terms of a *mantissa* and an *exponent*. For example, `0.0314159E+2` is the exponential-notation representation of 3.14159. (You see this by multiplying the mantissa, 0.0314159, by $10^2$, which is 10 raised to the power of the exponent, `2`.) Another example is `3e-6`, which denotes .000003 (that is, 3 times $1/10^6$)—it doesn't matter whether the `E` is upper or lower case.

Mathematicians and engineers who work with doubles often require operations such as exponentiation, square root, sine, cosine, and so on. The Java designers have assembled a class, named `Math`, that contains these methods and many more. For example, to compute the square root of a double, `D`, you need only say, `Math.sqrt(D)`:

```
double num = 2.2;
System.out.println("The square root of " + num + " is " + Math.sqrt(num));
```

(You do not have to construct a new `Math` object to use `sqrt`.) Similarly, `Math.pow(D1, D2)` computes $D1^{D2}$. Finally, `Math.abs(D)` computes and returns the *absolute value* of `D`, that is, `D` without its negation, if there was one. A list of the more useful computational methods for numbers appears in the supplementary section, "Helper Methods for Mathematics," which appears at the end of this chapter.

Although doubles and integers coexist reasonably well, complications arise when numbers of inappropriate data types are assigned to variables. First, consider this example:

```
int i = 1;
double d = i;
```

What value is saved in `d`'s cell? When `i` is referenced in `d`'s initialization statement, the integer 1 is the result. But 1 has type `int`, not `double`. Therefore, 1, is *cast* into

the fractional number, 1.0 (which has type `double`), and the latter is saved in `d`'s cell. (Computers use different forms of binary codings for integers and doubles, so there is a true internal translation from 1 to 1.0.)

In contrast, the Java compiler refuses to allow this sequence,

```
double d = 1.5;
int i = d + 2;
```

because the compiler determines that the result of `d + 2`, whatever it might be, will have data type `double`. Since a double will likely have a non-zero fractional part, there would be loss of information in the translation of the double (here, it would be 3.5), into an integer. The Java compiler announces that there is a *data-type error* in `i`'s initialization statement.

If the programmer truly wants to lose the fractional part of 3.5, she can use a *cast expression* to indicate this:

```
double d = 1.5;
int i = (int)(d + 2);
```

The phrase, `(int)` forces the double to be truncated into an integer; here, 3 is saved in `i`'s cell. We will have occasional use for such casts.

### Exercises

1. Rewrite the program, `CelsiusToFahrenheit`, into a program, `FahrenheitToCelsius`, that converts a double value of Fahrenheit degrees into double value of Celsius degrees and prints both values.

2. Recall that one kilometer is 0.62137 of a mile. Rewrite the program, `CelsiusToFahrenheit`, into a program, `KilometersToMiles`, that takes as input an integer value of kilometers and prints as output a double value of the corresponding miles.

3. Calculate the answers to these expressions; be careful about the conversions (casts) of integer values to doubles during the calculations:

   (a) `(5.3 + 7) / 2.0`

   (b) `(5.3 + 7) / 2`

   (c) `5.3 + (7 / 2)`

   (d) `(1.0 + 2) + ((3%4)/5.0)`

## 3.4 Booleans

The Java language lets you compute expressions that result in values other than numbers. For example, a calculation whose answer is "true" or "false" is a *boolean* answer; such answers arise when one compares two numbers, e.g., `6 > (4.5 + 1)` asks if 6 is greater than the sum of 4.5 and 1; the expression calculates to `true`, and this statement,

```
System.out.println( 6 > (4.5 + 1) );
```

prints `true`.

The Java data type, `boolean`, consists of just the two values, `true` and `false`.

Booleans are used as answers to questions; here is an example: We employ the temperature conversion formula to determine whether a given Celsius temperature is warmer than a given Fahrenheit temperature. The algorithm is

1. Set the given Celsius and Fahrenheit temperatures.
2. Convert the Celsius amount into Fahrenheit.
3. Compare the converted temperature to the other Fahrenheit temperature.

The corresponding Java statements read this simply:

```
double C = 22.0;
double F = 77.0;
double C_converted = ((9.0/5.0) * C) + 32;
System.out.print(C + " Celsius warmer than " + F + " Fahrenheit?  ");
System.out.println(C_converted > F);
```

These statements will print

```
22.0 Celsius warmer than 77.0 Fahrenheit?  false
```

Here are the standard comparison operations on numbers. All these operations use operands that are integers or doubles and all compute boolean results:

| Operation | Operator symbol | Example |
|---|---|---|
| greater-than | `>` | `6 > (3 + 1)` calculates to `true` |
| less-than | `<` | `6.1 < 3` calculates to `false` |
| less-than-or-equals | `<=` | `(3.14 * -1) <= 0` calculates to `true` |
| greater-than-or-equals | `>=` | `1 >= 2` calculates to `false` |
| equality | `==` | `(1 + 2) == (2 + 1)` calculates to `true` |
| inequality | `!=` | `0.1 != 2.1` calculates to `true` |

It is perfectly acceptable to save a boolean answer in a variable:

```
boolean b = (3 < 2);
```

This declares `b` and initializes it to `false`. Indeed, the same action can be done more directly as

```
boolean b = false;
```

because you are allowed to use the literals, `false` and `true` as well. You can reassign a new value to a boolean variable:

```
b = ((3 * 2) == (5 + 1));
```

This resets `b` to `true`. Notice that arithmetic equality in the above example is written as `==`. Also, arithmetic operations are calculated before comparison operations, which are calculated before the assignment, so the previous example can be abbreviated to

```
b = (3 * 2 == 5 + 1);
```

or just to

```
b =  3 * 2 == 5 + 1;
```

Finally, the usual numerical operations (`+, -, *, / , %`) cannot be used with boolean arguments, because it makes no sense to "add" or "subtract" logical truths. (E.g., what would `true - false` mean?) The Java compiler will report an error message if you attempt such actions.

### Exercises

1. Build a complete Java application that uses the technique in this section to determine whether a given celsius temperature is warmer than a given fahrenheit temperature. Test the application with 40 Celsisus and 40 Fahrenheit.

2. Build a Java application that is told a quantity of dimes and a quantity of nickels and prints whether the quantity of dimes is worth less than the quantity of nickels. Test the application with 4 dimes and 6 nickels.

3. Calculate the results of these expressions:

   (a) `(3 * 2) >= (-9 - 1)`

   (b) `3 * 2 != 5.5 + 0.4 + 0.1`

## 3.5 Operator Precedences

When the Java interpreter calculates an arithmetic expression, it proceeds from left to right: Given an expression of the form, `EXPRESSION1 operator EXPRESSION2`, the interpreter evaluates `EXPRESSION1` to its result before it evaluates `EXPRESSION2` and then applies the `operator` to the two resulting values.

Parentheses within an expression are highly recommended to make clear which expressions belong as operands to which operators. Consider `1 * 2 + 3`–what is the `*` operator's second operand? Is it `2`? Or `2 + 3`? If the expression was properly parenthesized, as either `1 * (2 + 3)` or `(1 * 2) + 3`, this confusion would not arise.

If parentheses are omitted from an expression that contains multiple operators, the Java compiler will apply its own rules to direct the computation of the Java interpreter. These rules, called *operator precedences*, are listed shortly.

One simple and "safe" example where parentheses can be omitted is a sequence of additions or multiplications, e.g., `1 + 2 + 3` or `1 * 2 * 3`. There is no confusion here, because both addition and multiplication are *associative*—the order that the additions (multiplications) are performed does not affect the result.

When multiplications and divisions are mixed with additions and subtractions in an expression without parentheses, the interpreter still works left to right, but when there is a choice, the interpreter does multiplications and divisions before additions and subtractions. We say that the *precedences* of multiplication and division are *higher* than those of addition and subtraction. For example, `1 + 2.0 * 3` results in `7.0`, because there is a choice between computing `1 + 2.0` first or `2.0 * 3` first, and the Java interpreter chooses the latter, that is, the multiplication is performed just as if the expression was bracketed as `1 + (2.0 * 3)`.

Here is the ordering of the precedences of the arithmetic operations, in the order from highest precedence (executed first, whenever possible) to lowest (executed last):

| |
|---|
| unary negation, e.g., `-3` |
| multiplication, `*`, division/quotient, `/`, and modulo `%` |
| addition/string concatenation, `+`, and subtraction, `-` |
| comparison operations, `<, <=, >, >=` |
| comparisons of equality, `==`, and inequality, `!=` |

Although it is not recommended to write expressions like the following one, the precedence rules let us untangle the expression and compute its answer:

```
    3 + -4 * 5 != 6 - 7 - 8
=>  3 + -20 != 6 - 7 - 8
=>  -17 != 6 - 7 - 8
=>  -17 != -1 - 8
=>  -17 != -9
=>  true
```

It is always best to use ample parentheses to indicate clearly the match of operands to operators in an expression.

**Exercises**

1. Calculate the answers for each of these expressions:
   (a) `6 * -2 + 3 / 2 - 1`
   (b) `5.3 + 7 / 2 + 0.1`
   (c) `3*2%4/5.0*2*3`
2. Another way of understanding operator precedences is that the precedences are rules for inserting parentheses into expressions. For example, the precedences tell us that the expression `1 + 2.0 * 3` should be bracketed as `1 + (2.0 * 3)`. Similarly, `5 - 3 / 2 * 4 + 6 * -2 / -3 + 1.5` is bracketed as `((5 - ((3 / 2) * 4)) + ((6 * -2) / -3)) + 1.5`. For each of the expressions in the previous exercise, write the expression with its brackets.

## 3.6 Strings, Characters, and their Operations

In Chapter 2 we noted that a textual phrase enclosed in double quotes, e.g., `"hello"`, is a *string*. Like numbers and booleans, strings are values that can be computed upon and saved in variables. Java's data-type name for strings is `String` (note the upper-case `S`); here is an example:

```
String name = "Fred Mertz";
```

Literal strings can contain nonletters, such as blanks, numerals, tabs, and backspaces; here are examples:

- `" "` (all blanks)
- `"1+2"` (two numerals and a plus symbol—*not* an addition!)
- `""` (an empty string)
- `"!?,.\t\b\'\"\n\r\\"` (four punctuation symbols, followed by
  - a tab (`\t`),
  - a backspace (`\b`),
  - a single quote (`\'`),
  - a double quote (`\"`),
  - a newline (`\n`),

  - a return (\r),
  - and a backslash (\\))

The last example shows that some symbols which represent special keys on the keyboard must be typed with a leading backslash. If you print the last string:

```
System.out.println("!?,.\t\b\'\"\n\r\\");
```

you will see

```
!?,.    '"
\
```

because the tab, backspace, quotes, newline, return, and backslash display themselves correctly in the command window.

As we saw earlier in this chapter, the `+` operator computes on two strings by concatenating (appending) them together:

```
System.out.println("My name is " + name);
```

The operator can also force a number (or boolean) to be concatenated to a string, which is a useful convenience for simply printing information:

```
System.out.println("My name is R2D" + (5 - 3));
```

Of course, the parenthesized subtraction is performed *before* the integer is attached to the string.

Integers and doubles are different from strings—you can multiply and divide with integers like `49` and `7` but you cannot do arithmetic on the the strings `"49"` and `"7"`. Indeed, `"49" + "7"` computes to `"497"`, *because the* `+` *symbol represents concatenation when a string argument is used.*

Later in this Chapter, we will learn techniques that transform numbers into strings and strings into numbers; these techniques will be necessary for an application to accept input data from a human user and do numerical computation on it.

The `String` type owns a rich family of operations, and we present several of them here. The operations for strings have syntax different from the arithmetic and comparison operators previously seen because strings are actually objects in Java, and the operations for strings are actually methods. For this reason, the following examples should be carefully studied:

- To compare two strings `S1` and `S2` to see if they hold the same sequence of characters, write

  ```
  S1.equals(S2)
  ```

For example, `"hello".equals("hel"+"lo")` returns the boolean answer, `true`, whereas `"hello".equals("Hello")` results in false.

Of course, the method can be used with a string that is named by a variable:

```
String s = "hello";
System.out.println(s.equals("hel"+"lo"));
System.out.println(s.equals(s));
```

prints `true` on both occasions.

- To determine a string's length, use the `length()` method: for string `S`, the message

```
S.length()
```

returns the integer length of the string, e.g.,

```
String s = "ab";
int i = s.length();
```

assigns 2 to `i`.

- For string, `S`, `S.trim()` computes a new string that looks like string `S` but without any leading or trailing blanks. For example,

```
String s = " ab c  ";
String t = s.trim();
```

assigns `"ab c"` to `t`—both the leading and trailing blanks are removed.

These methods and many others are listed in Table 5. Use the Table for reference; there is no need to study all the methods at this moment. In addition to the operations in the Table, you can locate others in the API documentation for class `String` within the `java.lang` package.

Finally, a warning: *Do not use the* `==` *operation to compare strings for equality!* For reasons that cannot be explained properly until a later chapter, the expression `S1 == S2` does not validate that strings `S1` and `S2` contain the same sequence of characters; instead, it checks whether `S1` and `S2` are the same identical object. The proper expression to check equality of two strings is `S1.equals(S2)`.

Figure 3.5: methods on strings

| Method | Semantics | Data typing restrictions |
|---|---|---|
| `S1.equals(S2)` | equality comparison—returns whether strings `S1` and `S2` hold the same sequence of characters | `S1` and `S2` must have type `String`; the answer has type `boolean`. |
| `S.length()` | returns the length of string `S` | if `S` has data type `String`, then the result has type `int` |
| `S.charAt(E)` | returns the character at position `E` in `S` | if `S` has data type `String` and `E` has data type `int`, then the result has data type `char` |
| `S.substring(E1, E2)` | returns the substring starting at position `E1` and extending to position `E2 - 1` | if `S` has data type `String`, `E1` and `E2` have data type `int`, then the result is a `String` |
| `S.toUpperCase()` | returns a string that looks like `S` but in upper-case letters only | if `S` has data type `String`, the result has data type `String` |
| `S.toLowerCase()` | returns a string that looks like `S` but in lower-case letters only | if `S` has data type `String`, the result has data type `String` |
| `S.trim()` | returns a string like `S` but without leading or trailing blanks | if `S` has data type `String`, the result has data type `String` |
| `S1.indexOf(S2, i)` | searches `S1` for the first occurrence of `S2` that appears inside it, starting from index `i` within `S1`. If `S2` is found at position `j`, and `j >= i`, then `j` is returned; otherwise, -1 is returned. | if `S1` and `S2` have data type `String` and `i` has data type `int`, the result has data type `int` |
| `S1.compareTo(S2)` | like the `equals` method, `compareTo` compares the characters in string `S1` to `S2`: if `S1` is "less than" `S2` according to the lexicographic (dictionary) ordering, -1 is returned; if `S1` and `S2` are the same string, 0 is returned; if `S1` is "greater than" `S2` according to the lexicographic ordering, then 1 is returned. | if `S1` and `S2` have data type `String`, then the result has data type `int` |

### Characters

The individual symbols within a string are called *characters*, and the data type of characters is `char`. A character can be represented by itself by placing single quotes around it, e.g, `'W'`, `'a'`, `'2'`, `'&'`, etc.

We noted earlier symbols like the tab and backspace keys have special codings:

- `\b` (backspace)
- `\t` (tab)
- `\n` (newline)
- `\r` (return)
- `\"` (doublequote)
- `\'` (single quote)
- `\\` (backslash)

Of course, a variable can be created to name a character:

```
char backspace = '\b';
```

Java uses the *Unicode* format for characters; inside the computer, characters are actually integers and you can do arithmetic on them, e.g., `'a'+1` converts (casts) `'a'` into its integer coding, 97, and adds 1.

*Begin Footnote:* Indeed, it is possible to cast an integer into a printable character: If one writes `(char)('a'+1)`, this casts the result of the addition into the character, `'b'`—the phrase, `(char)` is the cast, and it forces the numeric value to become a character value. *End Footnote.*

To examine a a specific character within a string, use the `charAt(i)` method from Table 5, where `i` is a nonnegative integer. (Important: The characters within a string, `S`, are indexed by the position numbers 0, 1, ..., `S.length()-1`.) Examples are:

- `char c = "abc".charAt(0)`

  which saves character `'a'` in variable `c`'s cell.

- ```
  String s = "abc" + "de";
  char c = s.charAt(2+1);
  ```

  which saves `'d'` in `c`'s cell.

- `"abc".charAt(3)`

  which results in an error that stops execution, because there is no character number 3 in `"abc"`.
  ```

Also, characters can be compared using the boolean operations in the previous section. For example, we can compare the leading character in a string, `s`, to see if it equals the character `'a'` by writing:

```
boolean result =  s.charAt(0) == 'a';
```

The comparision works correctly because characters are saved in computer storage as integer Unicode codings, so the equality, inequality, less-than, etc. comparisons can be performed.

**Exercises**

1. Calculate the results of these expressions:

   (a) `2 + ("a" + " ") + "bc"`

   (b) `1 + "" + 2 + 3`

   (c) `1 + 2 + "" + 3`

   (d) `1 + 2 + 3 + ""`

   (e) `1 + "" + (2 + 3)`

2. Here are two strings:

   ```
   String t = "abc ";
   String u = "ab";
   ```

   What is printed by these statements?

   (a) `System.out.println(t.equals(u));`

   (b) `System.out.println(u.charAt(1) == t.charAt(1));`

   (c) `System.out.println(t.length() - u.length());`

   (d) `System.out.println(u + 'c');`

   (e) `System.out.println(t.trim());`

   (f) `System.out.println(t.toUpperCase());`

## 3.7 Data-Type Checking

Because of the variety of values—integers, doubles, booleans, and strings—that can be expressed in a Java program, the Java compiler must enforce consist use of these values in statements; this is called *data-type checking*. For example, the second statement below is incorrect,

```
boolean b = true;
System.out.println(b * 5);
```

because the multiplication operator cannot execute with a boolean argument. Without doing the actual computation, the Java compiler notices the inconsistency and announces there is a *data-type error*.

A data type is a "species" of value, much like horses, lions, and giraffes are species of animals. Trouble can arise when different species mix, and for this reason, the Java compiler uses data types to monitor usage of values and variables for compatibility. More generally, data types are useful because

- They ensure compatibility of operands to operators, e.g., `1 * 2.3` is an acceptable expression but `"abc" * 2.3` is not.
- They predict a property about the result of an expression, e.g., whatever the result of `1 * 2.3` might be, it is guaranteed to be a `double` value.
- They help create variable cells of the appropriate format, e.g., `double d` declares a cell that is suited to hold only doubles.
- They help enfore compatibilities of expressions to variables in assignments, e.g., `d = 1 * 2.3` is an acceptable assignment, because it places a double into a cell declared to hold only doubles.

We can list the steps the Java compiler takes to type-check an expression; consider these two variables and the arithmetic expression that uses them:

```
int i = 1;
double d = 2.5;
 ... ((-3 * i) + d) > 6.1 ...
```

Since both `-3` and `i` have data type `int`, the Java compiler concludes that their addition will produce an answer that has type `int`.

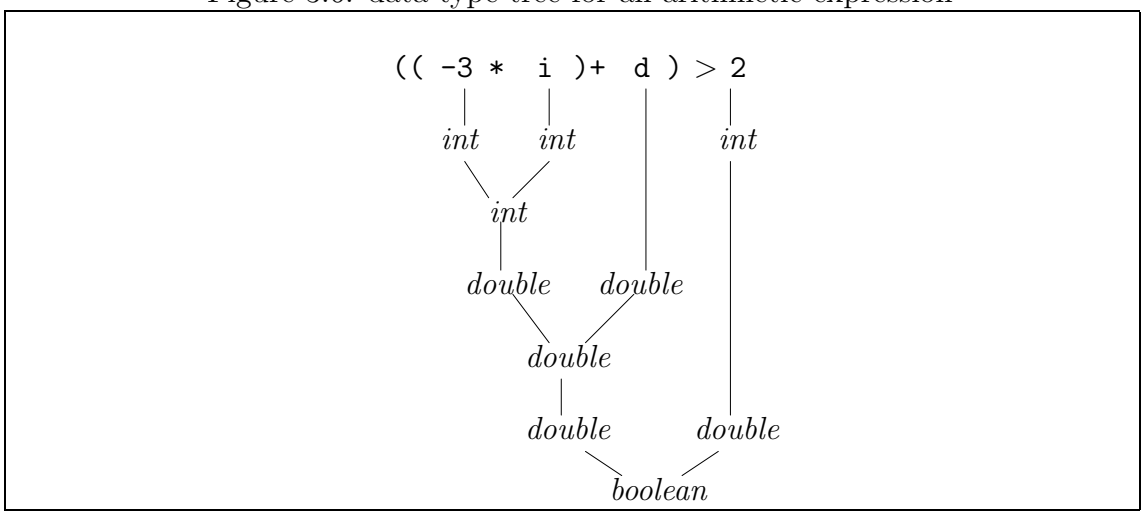
The process continues: Since `-3 * i` has type `int`, but `d` has type `double`, this implies that the left operand will be automatically cast into a double and that the answer for `(-3 * i) + 2`, must be a `double`. Finally, the less-than comparison will produce a value that has type `boolean`.

The process outlined in the previous paragraph is sometimes drawn as a tree, as in Figure 6. The tree displays the numbers' data types and how they combine into the type of the entire expression. A variant of this "data-type tree" is indeed drawn in primary storage by the Java compiler.

The notion of data type extends to objects, too—the Java compiler uses the name of the class from which the object was constructed as the object's data type. For example, in Figure 4 of Chapter 2 we wrote

```
GregorianCalendar c = new GregorianCalendar();
```

Figure 3.6: data-type tree for an arithmetic expression




which created a cell named `c` that holds (an address of) an object of data type `GregorianCalendar`. The Java compiler makes good use of `c`'s data type; it uses it to verify that object `c` can indeed be sent a `getTime` message, as in,

```
System.out.println(c.getTime());
```

This statement is well formed because `getTime` is one of the methods listed within `class GregorianCalendar`, and `c` has data type, `GregorianCalendar`. The Java compiler can just as easily notice an error in

```
c.println("oops");
```

because `c`'s data type is of a class that does *not* possess a `println` method.

In summary, there are two forms of data types in the Java language:

- *primitive types*, such as `int`, `double`, and `boolean`
- *reference (object) types*, such as `GregorianCalendar`

Variables can be declared with either form of data type.

Surprisingly, type `String` is a reference type in Java, which means that strings are in fact objects! This is the reason why the string operations in Table 5 are written as if they were messages sent to objects—they are!

### Exercises

Pretend you are the Java compiler.

1. *Without calculating the answers to these expressions*, predict the data type of each expression's answer (or whether there is a data-type error):

(a) `5.3 + (7 / 2)`

(b) `3 * (1 == 2)`

(c) `1 < 2 < 3`

(d) `"a " + 1 + 2`

(e) `("a " + 1) * 2`

2. Which of these statements contain data type errors?

```
int x = 3.5;
double d = 2;
String s = d;
d = (d > 0.5);
System.out.println(s * 3);
```

## 3.8 Input via Program Arguments

The applications in this chapter are a bit unsatisfactory, because each time we want to reuse a program, we must reedit it and change the values in the program's variable initialization statements. For example, if we wish to convert several temperatures from Celsius to Fahrenheit, then for each temperature conversion, we must reedit the program in Figure 4, recompile it, and restart it.

It would be far better to write the temperature conversion application just once, compile it just once, and let its user start the application again and again with different numbers for degrees Celsius. To achieve this, we make the application read a numeric temperature as *input data* each time it starts.

Java uses *program arguments* (also known as *command-line arguments*) as one simple way to supply input data to an application.

*Begin Footnote:* In the next Chapter, we learn a more convenient method for supplying input data. *End Footnote*

When she starts an application, the user types the program arguments—these are the inputs. The application grabs the program arguments and assigns them to internal variables.

Say that we have rewritten the temperature conversion program in Figure 4 to use program arguments:

- If you installed the JDK, then when you start an application, you type the program argument on the same line as the startup command, that is, you type

  ```
  java CelsiusToFahrenheit 20
  ```

  if you want to convert 20 Celsius to Fahrenheit.

- If you use an IDE to execute applications, you type the program argument in the IDE's field labelled "Program Arguments," "Arguments," or "Command-Line Parameters." (See Figure 7.) Then you start the application.

Before we develop the temperature-conversion application, we consider a simpler example that uses a program argument; perhaps we write an application, called `LengthOfName`, that accepts a person's name as its input data and prints the name's length in immediate response:

```
java LengthOfName "Fred Mertz"
The name, Fred Mertz, has length 10.
```

This is accomplished by this simple application, whose `main` method uses the name, `args[0]`, to fetch the value of the program argument:

```
/** LengthOfName prints the length of its input program argument */
public class LengthOfName
{ public static void main(String[] args)
  { String name = args[0];   // the program argument is held in  args[0]
    int length = name.length();
    System.out.println("The name, " + name + ", has length " + length);
  }
}
```

When an application with program arguments is started, the arguments are automatically placed, one by one, into distinct internal variables named `args[0]`, `args[1]`, and so on. Using these names, the `main` method can grab the program arguments.

In general, there can be no program arguments at all or multiple arguments. In the former case, nothing special needs to be done; in the latter, the arguments are written next to each other, separated by spaces, e.g.,

```
java ThisApplicationUsesFourArguments Fred Mertz 40 -3.14159
```

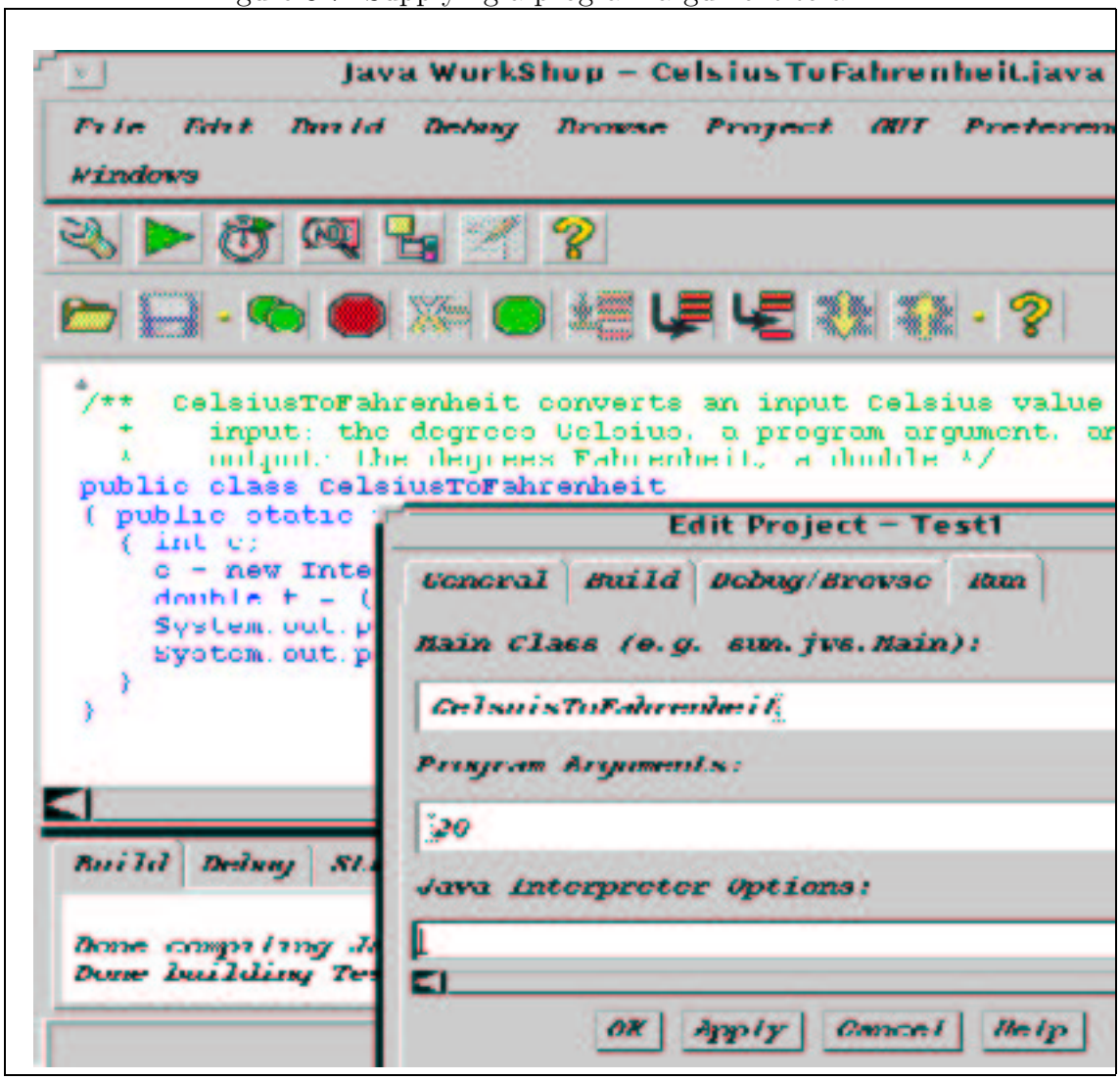
When the application is started, `args[0]` holds the string, `"Fred"`, `args[1]` holds the string, `"Mertz"`, `args[2]` holds the *string*, `"40"`, and `args[3]` holds the *string*, `"-3.14159"`—all the program arguments become strings, whether they are surrounded by double quotes or not!

Why are `args[2]` and `args[3]` in the example holding strings and not numbers? The answer lies within the phrase, `String[] args`, which appears in method `main`'s header line: This defines the name, `args`, for the variables that grab the program arguments, and the data type, `String`, which we *must* use with every `main` method, forces every program argument to become a string.

*Begin Footnote:* In Chapter 5 we learn that `args` is an example of a *parameter*, and in Chapter 8 we learn that `args` is an *array*. But we do not need the explanations of these concepts here to use program arguments. *EndFootnote*

Therefore, an application that relies on numbers for inputs must construct the numbers from the strings it reads as the program arguments. We now see how to do this.

Figure 3.7: Supplying a program argument to an IDE



### 3.8.1 Converting between Strings and Numbers and Formatting

With the aid of a "helper object," we can convert a string of numerals, like `"20"`, into the integer, 20. And, we can use another object to convert an integer or double into a nicely formatted string for printing.

#### Converting Strings into Integers and Doubles

The Java designers have written `class Integer`, from which we can construct an object that knows how to translate a string of numerals into an integer. (At this point, you should review the example in Figures 3 and 4, Chapter 2, that used `class GregorianCalendar` to create a similar "helper" object.)

Say that `S` is a string of numerals that we wish to translate into the corresponding integer, e.g.,

```
String S = "20";
```

Here are the steps we take:

- Create a new `Integer` object, by saying

  ```
  new Integer(S)
  ```

  The keyword, `new`, creates an object from `class Integer`. The object grabs the value of the argument (in this case, the value of `S` is `"20"`), saves it inside itself, and calculates its integer translation.

- Send the new object an `intValue` message. This can be done in two statements, e.g.,

  ```
  Integer convertor = new Integer(S);
  int c = convertor.intValue();
  ```

  or in just one:

  ```
  int c = new Integer(S).intValue();
  ```

  The message asks the object to return the integer translation. The integer—here, 20—is returned as the answer and is assigned to `c`.

In addition to `class Integer`, there is `class Double`, which creates objects that convert strings of numerals and decimals into double values, e.g.,

```
String approx_pi = "3.14159";
double d = new Double(approx_pi).doubleValue();
```

assigns 3.14159 to `d`.

*Begin Footnote:* Both `class Integer` and `class Double` are located in the Java package, `java.lang`. Although it is acceptable to add the statement, `import java.lang.*` to the beginning of an application that uses the classes, the Java interpreter will automatically search `java.lang` when it tries to locate a class; hence, `import java.lang.*` is an optional statement and will not be used here. *End Footnote*

### Converting a Number into a String and Formatting it for Printing

Converting a number into a string is simple: Because the operator, `+`, will combine a number and a string, we can concatenate an empty string to a number to "convert" it to a string:

```
double d = 68.8;
String s = d + "";
```

In the above example, `s` is assigned `"68.8"`.

When printing a double, we often wish to present it in a fixed decimal format. For example,

```
System.out.println ("$" + (100.0/3.0));
```

prints

```
$33.333333333333336
```

where we probably would prefer just `$33.33` to appear.

There is yet another class, named `DecimalFormat`, that constructs objects that know how to format doubles in a fixed decimal format:

- Because the class is located in the package, `java.text`, include

  ```
  import java.text.*;
  ```

  at the beginning of the application.

- Construct a `DecimalFormat` object as follows:

  ```
  new DecimalFormat(PATTERN)
  ```

  where `PATTERN` is a string that specifies the significant digits to the left and right of the decimal point. For example, the pattern, `"0.00"`, in `new DecimalFormat("0.00")` states that there must be *at least* one digit to the left of the decimal point and there must be *exactly* two digits to the right of the decimal point. (The rightmost fractional digit is rounded.)

  Other patterns, e.g., `".000"` or `"00.0"` are acceptable, and you may consult the Java API specification for `class DecimalFormat` for a full description of the legal patterns.

- Send a `format(D)` message to the object, where `D` is a double value, e.g.,

  ```
  DecimalFormat formatter = new DecimalFormat("0.00");
  double d = 100.0 / 3.0;
  String s = formatter.format(d);
  ```

  The `format` method returns the formatted string representation of its double argument, `d`; the previous statements place the string, `33.33`, in `s`'s cell.

Here are the steps presented together:

```
import java.text.*;
public class Test
{ public static void main(String[] args)
  {  ...
    DecimalFormat formatter = new DecimalFormat("0.00");
    double money = 100.0/3.0;
    System.out.println ("$" + formatter.format(money));
     ...
  }
}
```

### 3.8.2 Temperature Conversion with Input

We use the techniques from the previous section to make the temperature conversion application from Figure 4 read a Celsuis temperature as its input and display its translation to Fahrenheit. The input temperature will be a program argument that is supplied at the startup command, e.g.,

```
java CelsiusToFahrenheit 20
```

Within the application, the argument is fetched and converted to an integer by writing:

```
int c = new Integer(args[0]).intValue();
```

Recall that we use the name, `args[0]`—this is the internal variable that grabs the program argument when the program starts.

Next, we compute the result and format it as a decimal with exactly one fractional digit:

```
double f = ((9.0/5.0) * c) + 32;
DecimalFormat formatter = new DecimalFormat("0.0");
System.out.println("Degrees Fahrenheit = " + formatter.format(f));
```

Figure 3.8: application using a program argument

```
import java.text.*;
/**  CelsiusToFahrenheit converts an input Celsius value to Fahrenheit.
  *    input: the degrees Celsius, a program argument, an integer
  *    output: the degrees Fahrenheit, a double */
public class CelsiusToFahrenheit
{ public static void main(String[] args)
  { int c = new Integer(args[0]).intValue(); // args[0] is the program argument
    double f = ((9.0/5.0) * c) + 32;
    System.out.println("For Celsius degrees " + c + ",");
    DecimalFormat formatter = new DecimalFormat("0.0");
    System.out.println("Degrees Fahrenheit = " + formatter.format(f));
  }
}
```

Figure 8 shows the revised temperature-conversion program.

The program in Figure 8 can be used as often as we like, to convert as many temperatures as we like:

```
java CelsiusToFahrenheit 20

java CelsiusToFahrenheit 22

java CelsiusToFahrenheit -10
```

and so on—try it.

### An Execution Trace of Temperature Conversion

The concepts related to program arguments are important enough that we must analyze them in detail with an execution trace. Say that the user starts the application in Figure 8 with the program argument, `20`, which the Java interpreter reads as the string, `"20"`.

A `CelsiusToFahrenheit` object is created in computer storage, and its execution starts at the first statement of its `main` method:

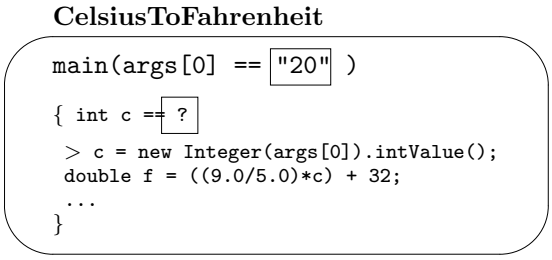
**CelsiusToFahrenheit**

```
main(args[0] == "20" )

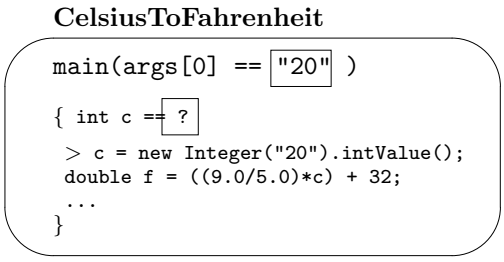
{ >int c = new Integer(args[0]).intValue();
 double f = ((9.0/5.0)*c) + 32;
 System.out.println("For Celsius degrees " + c + ",");
 DecimalFormat formatter = new DecimalFormat("0.0");
 System.out.println("Degrees Fahrenheit = " + formatter.format(f));
}
```

`args[0]` holds the program argument. (We will not depict the `System.out` object in these trace steps; just assume it is there.)

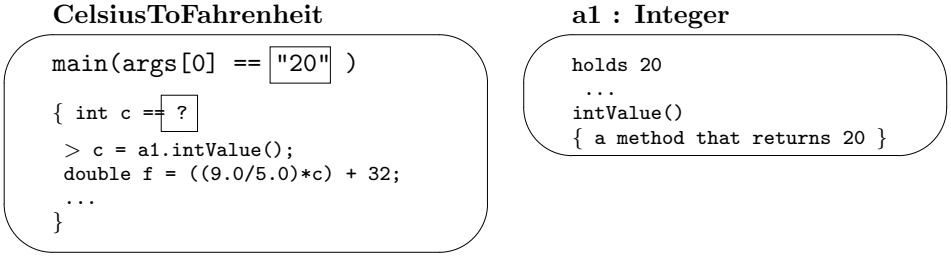
The first statement creates a cell named `c`; initially, there is *no* integer in it:




The value placed in the cell is computed by the expression, `new Integer(args[0]).intValue()`. First, `args[0]` must be calculated to its value; this is the string, `"20"`:

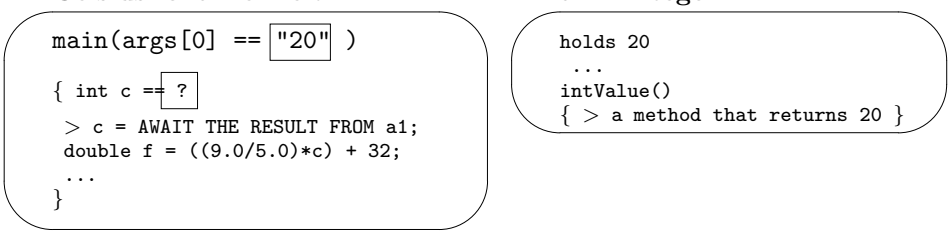



Next, the helper object is created in computer storage:




The computer invents an internal address for the helper object, say, `a1`, and inserts this address into the position where the object was created.

Next, execution of the statements in `main` pauses while object `a1` receives the message, `intValue()`, and executes the requested method:



Object `a1` quickly returns `20` to the client object:

**CelsiusToFahrenheit**

```
main(args[0] == "20" )

{ int c == ?
 > c = 20
 double f = ((9.0/5.0)*c) + 32;
 ...
}
```

(Note: Object `a1` stays the same, and we no longer show it.) Finally, the integer is assigned to `c`'s cell:

**CelsiusToFahrenheit**

```
main(args[0] == "20" )

{ int c == 20
 > double f = ((9.0/5.0)*c) + 32;
 System.out.println("For Celsius degrees " + c + ",");
 DecimalFormat formatter = new DecimalFormat("0.0");
 System.out.println("Degrees Fahrenheit = " + formatter.format(f));
}
```

The remaining statements in the `main` method execute in the fashion seen in earlier execution traces; in particular, a `DecimalFormat` object is created in the same manner as the `Integer` object, and its `format` method returns the string, `"68.0"`, which is printed.

### Exercises

1. Compile and execute Figure 8 three times with the program arguments 20, 22, and -10, respectively.

2. Revise the change-making program in Figure 3 so that the dollars and cents values are supplied as program arguments, e.g.,

   ```
   java MakeChange 3 46
   ```

3. Revise either or both of the programs, `CelsiusToFahrenheit` and `KilometersToMiles`, that you wrote as solutions to the preceding Exercises set, "Arithmetic with Fractions: Doubles," so that the programs use program arguments.

4. Say that a program, `Test`, is started with the program arguments `12 345 6 7.89`. Write initialization statements that read the arguments and place the string `"12"` into a variable, `s`; place the integer 6 into a variable, `i`; place the double 7.89 into a variable, `d`.

5. Write a sequence of three statements that do the following: reads a program argument as an integer, `i`; reads a program argument as an integer, `j`; prints `true` or `false` depending on whether `i`'s value i s greater than `j`'s.

6. Exercise 3 of the section, "Named Quantities: Variables," had difficulties printing correctly formatted answers when the dollars-cents amount printed contained a cents amount less than 10. Use `class DecimalFormat` to solve this problem. Test your solution with the inputs of four quarters, one nickel and one penny.

## 3.9 Diagnosing Errors in Expressions and Variables

Expressions and variables give a programmer more power, but this means there is more opportunity to generate errors. The errors can have these two forms:

- Errors related to spelling, grammar, and context (data types), that the Java compiler detects. These are called *compile-time errors*.

- Errors that occur when the program executes, and a statement computes a bad result that halts execution prematurely. These are called *run-time errors* or *exceptions*.

As we noted in the previous Chapter, the Java compiler performs a thorough job of checking spelling and grammar. For example, perhaps Line 4 of our program was written

```
System.out.println( (1+2(*3 );
```

The compiler notices this and announces,

```
Test.java:4: ')' expected.
   System.out.println( (1+2(*3 );
                              ^
```

Sometimes the narrative (here, `')' expected`) is not so enlightening, but the identification of the error's location is always helpful.

When using variables, take care with their declarations; one common error is misspelling the data-type name, `String`:

```
Test1.java:5: Class string not found in type declaration.
   string s;
   ^
```

The error message is not so helpful, but the location of the error, is the tip-off.

With the inclusion of arithmetic expressions, a new form of error, a *data-type error*, can appear. For example, the statement,

```
System.out.println(3 + true);
```

generates this compiler complaint,

```
Test.java:4: Incompatible type for +. Can't convert boolean to int.
   System.out.println(3 + true);
                        ^
```

which states that integers and booleans cannot be added together. Although such error messages are an annoyance initially, you will find that they are a great assistance because they locate problems that would certainly cause disaster if the program was executed.

Data-type errors also arise if a programmer attempts to save a value of incorrect type in a variable, e.g.,

```
int i = true;
```

The compiler will refuse to allow the assignment. The reason should be clear—if the next statement in the program was

```
System.out.println(i * 2);
```

then a disaster would arise. A variable's data type is a "guarantee" that the variable holds a value of a specific format, and the Java compiler ensures that the guarantee is maintained.

A related issue is whether a variable holds any value at all; consider this example:

```
int i;
System.out.println(i * 2);
```

If allowed to execute, these statements would cause disaster, because `i` holds no number to multiply by 2. The Java compiler notices this and complains.

Yet another variable-related error is the declaration of one name in two declarations:

```
int a = 1;
double a = 2.5;
System.out.println(a);  // which  a?
```

The compiler refuses to allow such ambiguities and announces a redeclaration error.

The previous diagnoses by the compiler are a great help, because they prevent disasters that would arise if the program was executed. Unfortunately, the Java compiler cannot detect all errors in advance of execution. For example, divison by zero is a disaster, and it is hidden within this sequence of statements:

```
int i = 0;
System.out.println(1 / i);
```

These statements are spelled correctly and are correctly typed, but when they are executed, the program stops with this announcement:

```
java.lang.ArithmeticException: / by zero
        at Test.main(Test.java:4)
```

This is a run-time error, an *exception*. The message identifies the line (here, Line 4) where the error arises and explains, in somewhat technical jargon, what the error is.

The above example of an exception is a bit contrived, but exceptions readily arise when a program's user types incorrect input information as a program argument. Consider this program sequence:

```
int i = new Integer(args[0]).intValue();
System.out.println(1 / i);
```

If the program's user enters `0` as the program argument, the program will halt with the same exception as just seen—the program is properly written, but its user has caused the run-time error. Even worse, if the user submits a non-number for the input, e.g., `abc`, this exception is generated:

```
java.lang.NumberFormatException: abc
        at java.lang.Integer.parseInt(Integer.java)
        at java.lang.Integer.<init>(Integer.java)
        at Test.main(Test.java:4)
```

The error message notes that the exception was triggered at Line 4 and that the origin of the error lays within the `Integer` object created in that line.

Finally, some errors will not be identified by either the Java compiler and interpreter. For example, perhaps you wish to print whether the values held by variables `x` and `y` are the same, and you write:

```
int x = 3;
int y = 7;
System.out.println(x = y);
```

The statements pass the inspection of the Java compiler, and when you start the application, you expect to see either `true` or `false` appear. Instead, you will see `7`!

The reason for this surprising behavior is that `x = y` is an *assignment* and not a test for equality (`x == y`). The statement, `System.out.println(x = y)`, assigns `y`'s value to `x`'s cell and then prints it. For historical reasons, the Java compiler allows assignments to appear where arithmetic expressions are expected, and results like the one seen here are the consequence. *Remember to use* `==` *for equality comparisons.*

## 3.10 Java Keywords and Identifiers

At the beginning of this chapter we noted that a variable name is an *identifier*. Stated precisely, an identifier is a sequence of letters (upper and lower case), digits, underscore symbol, `_`, and dollar sign, `$`, such that the first character is nonnumeric. Thus, `x_1` is an identifier, but `1_x` is not.

Identifiers are used to name variables, of course, but they are used for naming classes and methods also, e.g., `Total` and `println` are also identifiers and must be spelled appropriately. For consistent style, classes are named by identifiers that begin with an upper-case letter, whereas method names and variable names are named by identifiers that begin with lower-case letters.

Java's *keywords* have special meaning to the Java compiler and cannot be used as identifiers. Here is the list of keywords: `abstract boolean break byte case cast catch char class const continue default do double else extends final finally float for future generic goto if implements import inner instanceof int interface long native new null operator outer package private protected public rest return short static super switch synchronized this throw throws transient try var void volatile while`

## 3.11 Summary

Here is a listing of the main topics covered in this chapter:

**New Constructions**

- *variable declaration* (from Figure 1):

```
int quarters = 5;
```

- *arithmetic operators* (from Figure 1):

```
System.out.println( (quarters * 25) + (dimes * 10)
                      + (nickels * 5) + (pennies * 1) );
```

- *assignment statement* (from Figure 2):

```
money = money % 25;
```

- *data types* (from Figure 4):

```
int c = 22;
double f = ((9.0/5.0) * c) + 32;
```

- *program argument* (from Figure 6):

```
int c = new Integer(args[0]).intValue();
```

### New Terminology

- *operator*: a symbol, like `*` or `-`, that denotes an arithmetic operation
- *operand*: an argument to an operator, e.g., `i` in `3 + i`.
- *data type*: a named collection, or "species" of values. In Java, there are two forms, *primitive types*, like `int`, `double`, and `boolean`, and *reference types*, like `GregorianCalendar` and `DecimalFormat`.
- *variable*: a cell that holds a stored primitive value (like an integer) or an address of an object.
- *referencing a variable*: fetching a copy of the value saved in a variable
- *declaration*: the statement form that constructs a variable
- *initialization*: the statement that gives a variable its first, initial value.
- *assignment*: the statement form that inserts a new value into a variable
- *program argument*: input data that is supplied to a Java application at startup, typically by typing it with the startup command.
- *compile-time error*: an error that is identified by the Java compiler
- *run-time error (exception)*: an error that is identified by the Java interpreter, that is, when the application is executing.

### Points to Remember

- The Java language lets you calculate on numbers, booleans, strings, and even objects. These values are organized into collections—data types—so that the Java compiler can use the types to monitor consistent use of variables and values in a program.
- Input data that is supplied by means of program arguments are accepted as strings and are saved in the variables, `args[0]`, `args[1]`, and so on, in the `main` method.
- Strings can be converted into numbers, if needed, by means of objects created from `class Integer` and `class Double`.

**New Classes for Later Use**

- `class Integer`. Converts strings of numerals into integers; has method `intValue()`, which returns the integer value of the string argument, `S`, in `new Integer(S)`. See Figure 6.

- `class Double`. Converts strings of numerals into integers; has method `doubleValue()`, which returns the double value of the string argument, `S`, in `new Double(S)`.

- `String`. Used like a primitive data type, but it is actually a class; see Table 5 for an extensive list of operations (methods).

- `class DecimalFormat`. Found in package `java.text`. Has the method, `format(D)`, which returns the decimal-formatted value of double, `D`, based on the pattern, `P`, used in `new DecimalFormat(P)`. See Figure 8.

## 3.12 Programming Projects

1. One ounce equals 28.35 grams. Use this equivalence to write an application that converts a value in ounces to one in grams; an application the converts a value in kilograms to one in pounds. (Recall that one kilogram is 1000 grams, and one pound is 16 ounces.)

2. One inch equals 2.54 centimeters. Use this equivalence to write the following applications:

   (a) One that converts an input of feet to an output of meters; one that converts meters to feet. (Note: one foot contains 12 inchs; one meter contains 100 centimeters.)

   (b) One that converts miles to kilometers; one that converts kilometers to miles. (Note: one mile contains 5280 feet; one kilometer contains 1000 meters.)

   (c) The standard formula for converting kilometers to miles is `1 kilometer = 0.62137 miles`. Compare the results from the application you just wrote to the results you obtain from this formula.

3. Write an application that reads four doubles as its input. The program then displays as its output the maximum integer of the four, the minimum integer of the four, and the average of the four doubles. (Hint: the Java method `Math.max` (resp., `Math.min`) will return the maximum (resp., minimum) of two doubles, e.g., `Math.max(3.2, -4.0)` computes `3.2` as its answer.)

4. The two roots of the quadratic equation $ax^2 + bx + c = 0$ (where `a`, `x`, `b`, and `c` are doubles) are defined

$$x = \frac{-b + \sqrt{b^2} - 4ac}{2a} \text{ and } x = \frac{-b - \sqrt{b^2} - 4ac}{2a}$$

Write an application that computes these roots from program arguments for `a`, `b`, and `c`. (Hint: the square root of `E` is coded in Java as `Math.sqrt(E)`.)

5. (a) The compounded total of a principal, `p` (double), at annual interest rate, `i` (double), for `n` (int) years is

$$total = p((1 + i))^n)$$

Write an application computes the compounded total based on program arguments for `p`, `i`, and `n`. (Hint: the power expression, $a^b$, is coded in Java as `Math.pow(a,b)`.)

(b) Rewrite the above equation so that it solves for the value of `p`, thereby calculating the amount of starting principal needed to reach `total` in `n` years at annual interest rate `i`. Write an application to calculate this amount.

(c) Rewrite the above equation so that it solves for the value of `i`, thereby calculating the interest rate (the so-called *discount rate*) needed to reach a desired `total` starting at `p` and taking `y` years. Write an application to calculate this interest rate.

6. (a) The annual payment on a loan of principal, `p` (double), at annual interest rate, `i` (double), for a loan of duration of `y` (int) years is

$$payment = \frac{(1 + i)^y * p * i}{(1 + i)^y - 1}$$

Write an application that computes this formula for the appropriate program arguments. (Hint: the power expression, $a^b$, is coded in Java as `Math.pow(a,b)`.)

(b) Given the above formula, one can calculate the amount of debt remaining on a loan after `n` years of payments by means of this formula:

$$debtAfterNyears = (p * z^n) - (payment * \frac{z^n - 1}{z - 1})$$

where `z = 1 + i`, and `payment` is defined in the previous exercise.

Extend the application in the previous exercise so that it also prints the remaining debt for the Years 0 through 5 of the calculated loan.

7. (a) Starting with a principal, `p`, the total amount of money accumulated after contributing `c` each year for `y` years, assuming an annual interest rate of `i`, goes as follows:

$$total = (p * (1 + i)^y) + (c * \frac{(1 + i)^{y+1} - (1 + i)}{i})$$

Write an application that computes `total` for the four input values.

(b) Given a total amount money saved, `total`, the annual annuity payment that can be paid out every year for a total of `z` years, and still retain a nonnegative balance, is

$$payment = \frac{total * (1 + i)^{z-1} * i}{(1 + i)^z - 1}$$

assuming `i` is the annual interest that is paid into the remaining money. Write an application that computes the formula given the three inputs.

8. The acceleration of an automobile is the rate of change of its velocity. The formula for computing acceleration is

$$acceleration = \frac{V_f - V_i}{t}$$

where $V_i$ is the automobile's initial velocity, $V_f$ is its finial velocity, and `t` is the elapsed time the vehicle took, starting from $V_i$, to reach $V_f$. (Note: to use the formula, the measures of velocity and time must be consistent, that is, if velocities are expressed in miles per hour, then time must be expressed in hours. The resulting accleration will be in terms of miles and hours.)

(a) Write an application that computes acceleration based on input arguments for $V_i$, $V_f$, and $t$.

(b) Write an application that computes the time needed for an automobile to reach a final velocity from a standing start, given as input value of the acceleration.

(c) Write an application that computes the final velocity an automobile reaches, given a standing start and input values of time and acceleration.

9. Say that an automobile is travelling at an acceleration, `a`. If the automobile's initial velocity is $V_i$, then the total distance that the automobile travels in time `t` is

$$distance = V_i t + (1/2)a(t^2)$$

(Again, the measures of values must be consistent: If velocity is stated in terms of miles per hour, then time must be stated in hours, and distance will be computed in miles.) Use this formula to write the following applications:

(a) The user supplies values for $V_i$, `a`, and `t`, and the application prints the value of `distance`.

(b) The user supplies values only for $V_i$ and `a`, and the application prints the values of `distance` for times 0, 1, 2, ...

10. Write an application, `CountTheSeconds`, that converts an elapsed time in seconds into the total number of days, hours, minutes, and leftover seconds contained in the time. For example, when the user submits an input of 289932, the program replies

```
289932 is
3 days,
8 hours,
32 minutes, and
12 seconds
```

11. Write an application, `DecimalToBinary`, that translates numbers into their binary representations. The program's input is a program argument in the range of 0 to 63. For example, when the user types an input of 22, ¡/pre¿ the program replies:

```
22 in binary is 010110
```

(Hint: Study the change-making program for inspiration, and use `System.out.print` to print one symbol of the answer at a time.)

12. Number theory tells us that a number is divisible by 9 (that is, has remainder 0) exactly when the sum of its digits is divisible by 9. (For example, 1234567080 is divisible by 9 because the sum of its digits is 36.)

Write an application that accepts an integer in the range -99,999 to 99,999 as its input and prints as its output the sum of the integer's digits and whether or not the integer is divisible by 9. The output might read as follows:

```
For input 12345,
the sum of its digits is 15.
Divisible by 9?  false
```

13. In the `Math` object in `java.lang` there is a method named `random`: using `Math.random()` produces a (pseudo-)random double that is 0.0 or larger and less than 1.0. Use this method to write a program that receives as input an integer, `n`, and prints as its output a random integer number that falls within the range of 0 to `n`.

14. Next, use `Math.random()` to modify the `CelsiusToFahrenheit` application in Figure 4 so that it each time it is started, it chooses a random Celsius temperature between 0 and 40 and prints its conversion, e.g.,

    ```
    Did you know that 33 degrees Celsuis
    is 91.4 degrees Fahrenheit?
    ```

15. Write an application, `Powers`, that takes as its input an integer, `n` and prints as output `n`'s powers from 0 to 10, that is, $n^0$, $n^1$, $n^2$, ..., $n^{10}$.

16. Write an application, `PrimeTest`, that takes as its input an integer and prints out which primes less than 10 divide the integer. For example, for the input:

    ```
    java PrimeTest 28
    ```

    The program replies

    ```
    2 divides 28? true
    3 divides 28? false
    5 divides 28? false
    7 divides 28? true
    ```

17. Write an application that computes a worker's weekly net pay. The inputs are the worker's name, hours worked for the week, and hourly payrate. The output is the worker's name and pay. The pay is computed as follows:

    - gross pay is hours worked multiplied by payrate. A bonus of 50% is paid for each hour over 40 that was worked.
    - a payroll tax of 22% is deducted from the gross pay.

18. Write an application that computes the cost of a telephone call. The inputs are the time the call was placed (this should be written as a 24-hour time, e.g., 2149 for a call placed a 9:49p.m.), the duration of the call in minutes, and the distance in miles of the destination of the call. The output is the cost of the call. Cost is computed as follows:

    - Basic cost is \$0.003 per minute per mile.
    - Calls placed between 8am and midnight (800 to 2400) are subject to a 20% surcharge. (That is, basic cost is increased by 0.2.)
    - A tax of 6% is added to the cost.

    The difficult part of this project is computing the surcharge, if any. To help do this, use the operation `Math.ceil(E)`, which returns the nearest integer value that is not less than `E`. For example, `Math.ceil(0.2359 - 0.0759)` equals 1.0, but `Math.ceil(0.0630 - 0.0759)` equals 0.0.

## 3.13 Beyond the Basics



*In these optional, supplemental sections, we examine topics that extend the essential knowledge one needs to program arithmetic.*

### 3.13.1 Longs, Bytes, and Floats

Almost all the computations in this text rely on just integers, doubles, booleans, and strings. But the Java language possesses several other primitive data types, and we survey them here.

The data type `int` includes only those whole numbers that can be binary coded in a single storage word of 32 bits—that is, numbers that fall in the range of -2 billion to 2 billion (more precisely, between $-2^{31}$ and $(2^{31}) - 1$). To work with integers that fall outside this range, we must use *long integers*. (The data type is `long`.) Create a long integer by writing a number followed by an `L`, e.g., `3000000000L` is a long 3 billion. Long variables are declared in the standard way, e.g.,

```
long x = 3000000000L;
```

Longs are needed because answers that fall outside the range of the `int` type are unpredictable. For example, write a Java program to compute `2000000000 + 1000000000` and compare the result to `2000000000L + 1000000000L` The former *overflows*, because the answer is too large to be an integer, and the result is nonsensical. If you plan to work with numbers that grow large, it is best to work with longs.

At the opposite end of the spectrum is the `byte` data type, which consists of those "small" integers in the range -128 to 127, that is, those integers that can be represented inside the computer in just one byte—eight bits—of space. For example, 75 appears inside the computer as the byte 01001011. (To make 75 a byte value, write `(byte)75`.) Byte variables look like this:

```
byte x = (byte)75;
```

One can perform arithmetic on byte values, but overflow is a constant danger.

Java also supports the data type `float` of fractional numbers with 8 digits of precision. A float is written as a fractional number followed by the letter, `F`. For example, `10.0F/3.0F` calculates to 3.3333333. A double number can be truncated to 8 digits of precision by casting it to a float, e.g.,

```
double d = 10/3.0;
System.out.println( (float)d );
```

Figure 3.9: data types

| Data Type name | Members | Sample values | Sample expression |
|---|---|---|---|
| `byte` | one-byte whole numbers, in range -128 to 127 | `(byte)3` `(byte)-12` | `(byte)3 + (byte)1` |
| `int` | one-word integers (whole numbers), in range -2 billion to 2 billion | `3, -2000, 0` | `3+(2-1)` |
| `long` | two-word integers (whole numbers) | `3000000000L, 3L` | `2000000000L + 1000000000L` |
| `float` | fractional numbers with 8 digits of precision | `6.1F, 3.0E5F,` | `(float)(10.0/3.0F)` |
| `double` | fractional numbers with 16 digits of precision | `6.1, 3.0E5, -1e-1` | `10.5 / -3.0` |
| `char` | single characters (two-byte integers) | `'a', '\n', '2'` | `'a' + 1` |
| `boolean` | truth values | `true, false` | `(6 + 2) > 5` |
| `String` | text strings | `"Hello to you!", "", "49"` | `"Hello " + "to"` |

displays `3.3333333`, which might look more attractive than `3.3333333333333335`, which would be displayed if the cast was omitted.

Floats can be written in exponential notation e.g.,

```
float x = 3e-4F;
```

which saves .0003F in `x`.

Table 9 summarizes the data types we have encountered.

### 3.13.2 Helper Methods for Mathematics

Within the `java.lang` package, `class Math` provides a variety of methods to help with standard mathematical calculations. (Some of the methods were mentioned in the Programming Projects section.) Table 10 lists the more commonly used ones; many more can be found in the Java API for `class Math`.

Figure 3.10: mathematical methods

| Constant name | Semantics | Data typing |
|---|---|---|
| `Math.PI` | the value of PI, 3.14159... | `double` |
| `Math.E` | the value of E, 2.71828... | `double` |
| Method | Semantics | Data typing |
| `Math.abs(E)` | returns the absolute value of its numeric argument, `E` | returns the same data type as its argument, e.g., a `double` argument yields a `double` result |
| `Math.ceil(E)` | returns the nearest integer value that is not less than `E`. (That is, if `E` has a non-zero fraction, then `E` is increased upwards to the nearest integer.) | `E` should be a `double`; the result is a `double` |
| `Math.floor(E)` | returns the nearest integer value that is not greater than `E`. (That is, if `E` has a non-zero fraction, then `E` is decreased downwards to the nearest integer.) | `E` should be a `double`; the result is a `double` |
| `Math.cos(E)` | returns the cosine of its argument, `E`, a numeric value in radians. (Note: 180 degrees equals PI radians.) | returns a `double` |
| `Math.max(E1, E2)` | returns the maximum of its two numeric arguments | Similar to multiplication—the result data type depends on the types of the arguments |
| `Math.min(E1, E2)` | returns the minimum of its two numeric arguments | Similar to multiplication—the result data type depends on the types of the arguments |
| `Math.pow(E1, E2)` | returns `E1` raised to the power `E2`: $E1^{E2}$ | Both arguments must be numeric type; the result is a `double` |
| `Math.random()` | returns a (pseudo)random number that is 0.0 or greater but less than 1.0 | the result has type `double` |
| `Math.round(E)` | returns `E` rounded to the nearest whole number | if `E` has type `float`, the result has type `int`; if `E` has type `double`, the result has type `long` |
| `Math.sin(E)` | returns the sine of its argument, `E`, a numeric value in radians. (Note: 180 degrees equals PI radians.) | returns a `double` |
| `Math.sqrt(E)` | returns the square root of its argument | The arguments must be numeric; the result is a `double` |
| `Math.tan(E)` | returns the tangent of its argument, `E`, a numeric value in radians. (Note: 180 degrees equals PI radians.) | returns a `double` |

### 3.13.3 Syntax and Semantics of Expressions and Variables

The syntax and semantics of expressions and variables are surprisingly rich, and we will reap rewards from investing some time in study. The starting point is the notion of "data type":

#### Data Type

Here is the syntax of Java data types as developed in this chapter:

```
TYPE ::=  PRIMITIVE_TYPE | REFERENCE_TYPE
PRIMITIVE_TYPE ::=  boolean | byte | int | long | char | float | double
REFERENCE_TYPE ::=  IDENTIFIER | TYPE[]
```

Types are either primitive types or reference types; objects have data types that are `REFERENCE_TYPE`s, which are names of classes—identifiers like `GregorianCalendar`. Chapter 2 hinted that `String[]` is also a data type, and this will be confirmed in Chapter 8.

*Semantics:* In concept, a data type names a collection of related values, e.g., `int` names the collection of integer numbers. In practice, the Java compiler uses data types to monitor consistent use of variables and values; see the section, "Data Type Checking" for details.

#### Declaration

```
DECLARATION ::=  TYPE IDENTIFIER [[ = INITIAL_EXPRESSION ]]? ;
```

A variable's declaration consists of a data type, followed by the variable name, followed by an optional initialization expression. (Recall that the `[[ E ]]?` states that the `E`-part is optional.) The data type of the `INITIAL_EXPRESSION` must be consistent with the `TYPE` in the declaration.

See the section, "Java Keywords and Identifiers," for the syntax of Java identifiers.

*Semantics:* A declaration creates a cell, named by the identifier, to hold values of the indicated data type. The cell receives the value computed from the initial expression (if any).

#### Statement and Assignment

The addition of declarations and assignments motivates these additions to the syntax rule for statements:

```
STATEMENT ::=  DECLARATION
             | STATEMENT_EXPRESSION ;
STATEMENT_EXPRESSION ::=  OBJECT_CONSTRUCTION | INVOCATION | ASSIGNMENT
```

The syntax of an assignment statement goes

```
ASSIGNMENT :=  VARIABLE = EXPRESSION
VARIABLE ::=  IDENTIFIER
```

The Java compiler enforces that the data type of the expression to the right of the equality operator is compatible with the type of the variable to the left of the equality.

*Semantics:* An assignment proceeds in three stages:

1. determine the cell named by the variable on the left-hand side of the equality;
2. calculate the value denoted by the expression on the right-hand side of the equality;
3. store the value into the cell.

### Expression

The variety of arithmetic expressions encountered in this chapter are summarized as follows:

```
EXPRESSION ::=  LITERAL
                | VARIABLE
                | EXPRESSION INFIX_OPERATOR EXPRESSION
                | PREFIX_OPERATOR EXPRESSION
                | ( EXPRESSION )
                | ( TYPE ) EXPRESSION
                | STATEMENT_EXPRESSION
LITERAL ::=  BOOLEAN_LITERAL
             | INTEGER_LITERAL | LONG_LITERAL
             | FLOAT_LITERAL | DOUBLE_LITERAL
             | CHAR_LITERAL | STRING_LITERAL
INFIX_OPERATOR ::=   +  |  -  |  *  |  /  |  %
                     |  <  |  >  |  <=  |  >=  |  ==  |  !=
PREFIX_OPERATOR ::=  -
INITIAL_EXPRESSION ::=  EXPRESSION
```

A `LITERAL` is a constant-valued expression, like `49`, `true`, or `"abc"`. The `( TYPE ) EXPRESSION` is a cast expression, e.g., `(int)3.5`.

*Semantics:* Evaluation of an expression proceeds from left to right, following the operator precedences set down in the section, "Operator Precedences."