CS 740 – Computational Complexity and Algorithm Analysis

Spring Quarter 2012

Slides 1

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Today's Session

1. A motivating example
2. What is *Computational Complexity* all about?
3. More examples
4. A computational complexity success story
5. Organizational matters

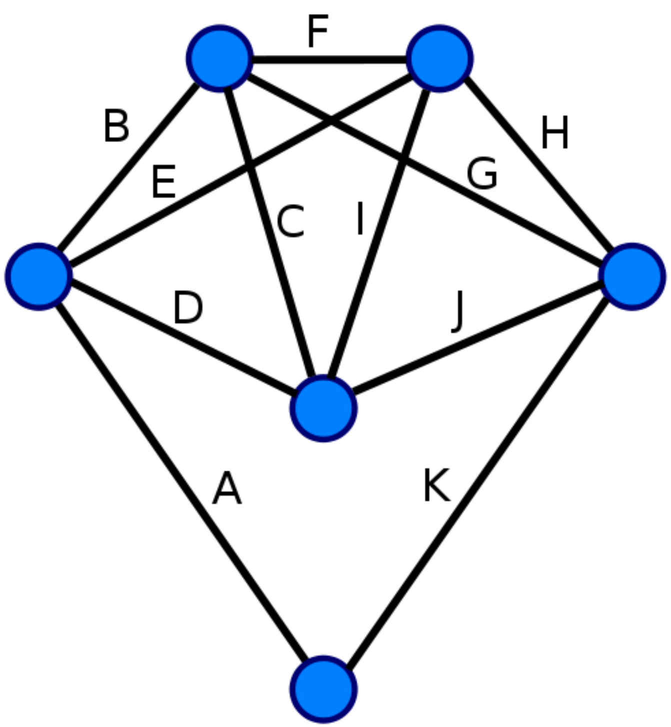
A motivating example

Input: A connected graph (undirected)

Output: “yes” if the graph has a path which

- **visits each edge exactly once and**
- **starts and ends on the same vertex.**

Output: “no” otherwise



Find an algorithm for this problem.

[Such graphs are called *Eulerian*.]

[Such paths are called *Eulerian cycles*.]

Naive algorithm 1 – brute force

- Graph consists of
  - m vertices: 1,2,...,m
  - n edges, written as (x,y) [edge between vertex x and vertex y]

```
for each permutation P of the n edges
      [i.e., P = ( (x1,y1),...,(xn,yn) )]
  output “yes” if P constitutes an Eulerian cycle
output “no” if no Eulerian cycle was found
```

Is this a good algorithm?

How to improve?

Naive algorithm 1 – brute force

- Graph consists of
  - m vertices: 1,2,...,m
  - n edges, written as (x,y) [edge between vertex x and vertex y]

```
for each permutation P of the n edges
      [i.e., P = ( (x_1,y_1),...,(x_n,y_n) )]
  output “yes” if P constitutes an Eulerian cycle
output “no” if no Eulerian cycle was found
```

How costly is this? (roughly, order of magnitude)

If no Eulerian cycle is found, we have to check n! permutations (that’s the *worst case*).

n! is quite a lot!

n	n!
5	120
10	3,628,800
15	$\approx 1.3 \cdot 10^{12}$
20	$\approx 2.4 \cdot 10^{18}$
50	$\approx 3 \cdot 10^{64}$
70	$\approx 10^{100}$

10^{100} – That's more than there are particles in the universe

n! is quite a lot!

• 10^1	
• 10^2	
• 10^3	Number of students in the college of engineering
• 10^4	Number of students enrolled at WSU
• 10^6	Number of people in Dayton
• 10^7	Number of people in Ohio Number of seconds in a year
• 10^8	Number of people in Germany
• 10^{10}	Number of stars in the galaxy Number of people on earth Number of milliseconds per year
• 10^{20}	Number of stars in the universe
• 10^{80}	Number of particles in the universe

Naive algorithm 2 - backtracking

- Graph consists of
  - m vertices: 1,2,...,m
  - n edges, written as (x,y) [edge between vertex x and vertex y]

```
Fix the first vertex, say, x1.
Make a systematic depth-first search on the graph
            edges.
  For each resulting maximal path P, if P is an
            Eulerian cycle, output “yes”.
If no Eulerian cycle is found, output “no”.
```

Algorithm is better – but is it *significantly* better?

In the worst case, fully connected graph, we have to check m! paths.

When do we know we have *the best* algorithm?

Smart algorithm

Theorem:

A connected undirected graph is Eulerian if and only if every vertex has an even number of edges (counting loops twice).

```
For each vertex x
   if number of edges of x is odd, output “No” and
                    stop.
Output “Yes”.
```

In the worst case, we have to make m checks, each of which consists of counting at most n edges.

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1. A motivating example
2. **What is *Computational Complexity* all about?**
3. More examples
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What is Comp. Complexity all about?

- Some problems seem hard but are not. Identify them.
- Some problems seem easy but are not. Identify them.

- Know when to stop searching for a smarter algorithm.
  [And instead turn to optimizations and heuristics.]

- What does “computationally hard problem” mean exactly?
- In what sense can we really say that some problem is computationally harder than some other problem?

What is Comp. Complexity all about?

- It’s a part of *theoretical* computer science.
- It’s a formal theory of the analysis of computational hardness of problems.
- It’s probably rarely going to help you directly in practice.
- But indirectly, in form of having a systematic understanding of problem hardness, it is indispensable.

What is Comp. Complexity all about?

We will certainly also learn about the

P = NP?

problem.

What it is.
Why it is important.

Why some people make such a fuzz about it.

Some basic notions

- **Problem:**
  A mapping from input to output.

- **Algorithm:**
  A method or a process followed to solve a problem.

- **A problem can have many algorithms.**

Some basic notions

- **Problem:**
  A mapping from input to output.

- **Algorithm:**
  A method or a process followed to solve a problem.

- **We focus on problems. Algorithm analysis is also interesting, but not as foundationally important.**

Some basic notions

- **Problem:**
  A mapping from input to output.
  - **We use the *order of magnitude* of the number of steps needed to solve a problem.**
    - **measured as a number which depends on the *input size*.**
  - **We are really interested in the *worst case* scenario.**
    - **i.e., how many steps do we need if the input is as unfavorable as possible?**

Can also be studied:

best case (usually not that interesting)

average case (of practical interest for concrete algorithms)

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Linear Search

- **Input:** **An array *A* of integers, and a value *v*.**
- **Output:** **"Yes" if *v* is an element of *A*;**
  "No" otherwise.

A =

3	12	7	25	7	32	11	56	28	43	6	87	68	91	2

v = 28

Algorithms: *Exhaustive search; random search; sort and linear search; sort and binary search*

Linear Search

Not all inputs of a given size take the same time to run.

Sequential search for *v* in an array of *n* integers:

- **Begin at first element in array and look at each element in turn until *v* is found**

Best case:

Worst case:

Average case?

Linear Search – Average case

Case: i	Time: T(i)	Probability P(i)	Cost: T(i) * P(i)
1	1	1/n	1/n
2	2	1/n	2/n
3	3	1/n	3/n
…	…	…	…
n	n	1/n	1

$$\begin{aligned}\sum Cost &= 1/n + 2/n + \cdots + n/n \\ &= \frac{1}{n} \times \sum_{i=1}^{n} i \\ &= \frac{1}{n} \times \frac{n(n+1)}{2} = \frac{n+1}{2}\end{aligned}$$

Bubble Sort

- **Algorithm 1:**

```
for pass = 0…n-1 {
  for position = 0…n-1 {
    if (array[position] > array[position+1]) {
      swap (array[position], array[position+1])
    }
  }
}
```

Best, worst, average: $\approx (n^2)$

Bubble Sort

- **Algorithm 2:**

```
for pass = 0…n-1 {
  for position = 0…n-pass-1 {
    if (array[position] > array[position+1]) {
      swap (array[position], array[position+1])
    }
  }
}
```

Best, worst, average: $\approx (n^2)$

(Within a constant factor)

Bubble Sort

- **Algorithm 3:**

```
for pass = 0…n-1 {
  swaps = 0;
  for position = 0…n-pass-1 {
    if (array[position] > array[position+1]) {
      swap (array[position], array[position+1]);
      swaps++;
    }
  if (swaps == 0) return;
  }
}
```

Best: $\approx n$,

Worst: $\approx (n^2)$

Average = ??

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A success story

- Research Area: Semantic Web
  Aimed at endowing information on the World Wide Web with "machine-processable meaning" (semantics).

- This is done using languages for representing knowledge.
  E.g., the knowledge on a website.
- These languages can also be used for querying this knowledge.

- These languages are also able to represent problems.
  Knowledge: A graph.
  Query: Does it have an Eulerian cycle?

- These languages differ in how "complex" the problems representable in them can be.

A success story

- Web Ontology Language OWL
  Recommended standard by the World Wide Web Consortium W3C.
  Established 2004, revised 2009.

- Research which led to OWL was driven by computational complexity analysis.

- Complexity used as a priori measure for runtime.

- Goal was finding a language which allows maximum freedom in specifying knowledge (problems), while being of minimal complexity.

- This approach paid off extremely well:
  Currently e.g. substantial commercial interest generated.

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Organizational Matters

- Office Hours: Wed 5:00-6:0pm, Joshi 389.
  Email contact preferred.
- Textbook (required):
  Thomas A. Sudkamp, Languages and Machines, Third Edition, Addison Wesley, 2006.
- Textbook (supplementary):
  Michael R. Garey and David S. Johnson, Computers and Intractability, Freeman, 1979
- Grading:
  Midterm exam: 30%
  Final exam: 50%
  Exercises: 20%

Distance learning option

- **This class has a distance learning option.**
- **I.e. there will be quite a bit of communication via email. If something slips my attention (e.g., concerning difficulties with the online material) please let me know.**
- **It is okay to send me solutions to homework assignments by email (scans or typeset [LaTeX recommended]). However, it is your responsibility that everything is clearly readable after printout. I will deposit the graded homework with the department secretary for pick-up [let me know if this is a problem].**
- **I assume everybody will be physically present for the exams. Information on exam dates and times is on the course website.**

Organizational matters

- I prefer to use a public website:
  http://knoesis.wright.edu/faculty/pascal/teaching/s12/complexity.html

- On the website, there is a link to last year's lecture with the old lecture manuscript [you can ignore this, but you may find it instructive to take a look].

- The new manuscript will be an updated/corrected version of last year's and will be posted shortly before class (usually, the evening before) in a near-final version. The final version will be posted, usually, on the evening of the class session.

- The manuscript bears a date (first page) and margin notes indicate the dates when which part was covered (important e.g. for due dates of exercises).

Organizational matters

- **I will be absent on several occasions: 05/28, 05/30.**
  - **To compensate:**
    - **all other classes 6:05pm to 7:30pm!**

- **We will frequently make exercise sessions.**
  You will get exercises marked "hand-in", to be done at home and graded by me, and discussed afterwards in class.

- **Each "hand-in" exercise counts 5 points. An average of 4 points is 100%.**
  Exercises are due *one week after I pose them – before class*.

- **The exercises are the tough part of the class. If you stay on top of them, you'll find the exams relatively easy.**

Course overview

Tentative

We recap most of chapter 8

We cover most of chapters 14 and 15

We cover parts of chapters 16 and 17, tbd what/how much.