Correctness of Slicing Finite State Machines

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What is Slicing?

Pick one or more program points of interest, called the \textit{slicing criterion}
What is Slicing?

Walk backwards to find the nodes (the slice set) that the nodes in the slicing criterion depend on
  ▶ through data dependence, or
  ▶ through control dependence
Remove nodes not in the slice set.
What is Slicing?

Applications include

- compiler optimizations
- debugging
- model checking
- protocol understanding
Outline of Talk

1. Summarize slicing of deterministic programs
   ▶ for various kinds of control flow graphs
   ▶ with focus on correctness properties
2. Discuss how to extend to non-determinism
   ▶ restate desired correctness properties
3. Application: extended finite state machines (EFSM)
   ▶ outline technical details
   ▶ sketch algorithm
   ▶ give example slices
Initial Assumptions

We assume for now a deterministic setting, and consider a control-flow graph (CFG) where nodes are either

- assignments
  - with one successor
  - to be replaced by skip if sliced away
- conditionals
  - with two successors
  - to be replaced by suitable goto if sliced away
- end node, with no successors

There may be a post-processing phase which

- may re-wire the CFG, removing skip nodes etc
- is not the focus of this work
Correctness Criteria

Correctness of slicing (early work: [Ball & Horwitz, 1993] and [Hatcliff et al, 2000]) may be phrased as simulation:

- the observables are the nodes in the slice set
- the equivalences are modulo relevant variables

Weak Correctness:

Each observable action by the original program can be simulated by the sliced program

In deterministic setting, this implies (if end observable)

Each observable action by the sliced program can be simulated by the original program unless original program does unobservable loop

Strong correctness: in addition to weak correctness,

Each observable action by the sliced program can be simulated by the original program
Basic Dependence Relations

It is standard to demand the slice set to be

- closed under data dependence
- closed under some kind of control dependence

Defining data dependence (DD) is non-controversial:

\[ b \text{ is data dependent on } a \text{ if there is a path from } a \text{ to } b, \text{ and a variable defined in } a \text{ and used in } b \text{ but not redefined in the interior of that path} \]

The proper notion of control dependence depends on

- the correctness criterion aimed for
- the kind of CFGs that are considered
Classical Definitions of Control Dependence

Assume the CFG has unique end node e. We say:

*b post-dominates a* iff
*all paths from a to e contain b*

*b is control dependent on a* if

- a is not strictly postdominated by b
- there is a path from a to b where all nodes except a are postdominated by b

This ensures weak correctness. To get strong correctness, use strong post-domination [Podgurski & Clarke]:

*b strongly post-dominates a* iff
*all maximal paths from a contain b*
For strong correctness, Ranganath et al proposed [ESOP’05 & TOPLAS’07] a conservative extension of the strong version of control dependence: 

*b* is NTSCD-control dependent on *a* iff from

- one of *a*’s successors, *b* cannot be avoided forever
- another of *a*’s successors, *b* may be avoided forever

This ensures strong correctness provided the CFG is reducible (forward edges form a DAG; for back edges, the target dominates the source)
To get strong correctness, slices must include all nodes that influence guards of potential loops

- great, if slicing to preserve liveness properties
- not so great, if slicing for program understanding

Hence we may want to go for weak correctness. For that, the relevant condition [Amtoft, IPL’08] is that the slice set should be closed under a ternary relation: $c \& b$ are weakly order dependent on $a$ iff

- path $[a..b] \not\ni c$ and path $[a..c] \not\ni b$
- $a$ has successor $x$ such that either $b$ is reachable from $x$ and all $[x..c]$ contain $b$, or $c$ is reachable from $x$ and all $[x..b]$ contain $c$.

**Conservative extension:** for a CFG with an end node which is part of slicing criterion, weak order dependence gives the same closure as standard control dependence.
Making Sense of Chaos

Danicic et al [TCS 2011] observed, in a setting that generalizes most previous frameworks for slicing, that

- the key to get weak correctness is to ensure that the slice set is weak commitment-closed (WCC): each node has at most one next observable
Correctness of Slicing

Finite State Machines

Torben Amtoft

Motivating Slicing

Deterministic Setting

Goal

Methods

Non-Determinism

Goal

Method

EFSM Development

Adapting Definitions

Slicing Algorithms

Conclusion

Making Sense of Chaos

Danicic et al [TCS 2011] observed, in a setting that generalizes most previous frameworks for slicing, that

- the key to get weak correctness is to ensure that the slice set is **weak commitment-closed (WCC)**: each node has at most one next observable

A conditional that is not in the slice...

Boundary representing nodes in the slice

First node in slice encountered along path of non-relevant nodes
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- the key to get weak correctness is to ensure that the slice set is weak commitment-closed (WCC): each node has at most one next observable

- the key to get strong correctness is to ensure that the slice set is strong commitment-closed (SCC): each node either has no next observable, or one next observable which no infinite path can miss
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Correctness for Non-Determinism, I

For weak correctness, our previous attempt

\[ \text{Each observable action by the original program can be simulated by the sliced program} \]

while still necessary does no longer suffice as it allows increased non-determinism, giving the sliced program freedom to do actions the original program would not do.

What in a deterministic version was implied by the above, we now need to explicitly state:

\[ \text{Each observable action by the sliced program can be simulated by the original program unless original program does unobservable loop or original program gets stuck} \]

with a line added to allow for no feasible choices
Correctness for Non-Determinism, II

We stated **Weak Correctness for Non-Determinism**: 

1. Each observable action by the original program can be simulated by the sliced program.
2. Each observable action by the sliced program can be simulated by the original program unless original program does unobservable loop or original program gets stuck.

If we disallow the removal of unobservable loops we get **Strong Correctness for Non-Determinism**:

1. Each observable action by the original program can be simulated by the sliced program.
2. Each observable action by the sliced program can be simulated by the original program unless original program gets stuck.

Choice (debatable?): slicing may remove “stuckness”.
Computing Slice Sets: Basic Approach

**Q:** which control dependencies are suitable for non-determinism?

**A:** we probably need to invent some quite sophisticated ones... but which???

What we shall require about the slice set is: not that is is closed under some kind of control dependence, but that

- it is closed under data dependence
- it satisfies WCC, and perhaps even SCC

We expect to be able to prove weak correctness from

**WCC:** no node has two “next observable”s

and to prove strong correctness from

**SCC:** each node either has no next observable, or one which no infinite path can miss

We shall now work out this agenda for a concrete setting.
To model reactive systems, an EFSM has

- a number of states
- labeled transitions between states

Each transition is triggered

- when guard is true
- possibly consuming event from environment
- possibly doing action on store
Extended Finite State Machines, Slicing

We want the slice set to contain transitions, rather than nodes, as this is where the real action takes place. If a transition is not part of the slice set its

- guard becomes true
- action becomes skip

Example: the slicing criterion $t_2$ does not depend on $t_1$

$t_1: [y > 0]/x := y$

and hence $t_1$ is an “$\epsilon$-transition” in the sliced program:

$t_1$: skip

which is less likely to be stuck than the original program.
How to modify definitions developed for CFGs?

*node a has node b as next observable if*

- there is a path from a to b of transitions not in slice set
- a transition from b belongs to the slice set

- the next observables of S1 are S2 and S3
- the slice set does thus *not* satisfy WCC
- and indeed, the sliced EFSM might do t6 while
  - the original EFSM can’t do t6 (due to guard for t3)
  - but may be able to do t5.
If the slice set is weakly committed then
- if the original EFSM can do an observable step it can be simulated by the sliced EFSM
- if the sliced EFSM can do an observable step then either
  1. it can be simulated by the original EFSM, or
  2. the original EFSM may get stuck, or
  3. the original EFSM may enter an unobservable loop
where (3) is ruled out if slice set strongly committed.
Finding Least Set Satisfying WCC: Theory

Observe: if WCC does not hold then the situation is:

\[ n \quad \overset{t_1}{\rightarrow} \quad n_1 \quad \overset{t_2}{\rightarrow} \quad n_2 \]

and \( t_1 \) will belong to any superset satisfying WCC.

For a given slicing criterion, there thus exists a least superset that satisfies WCC (and is closed under DD) and we can write an algorithm to iteratively find this set:

- from the observables, do a backwards breadth-first search through transitions not in slice set.
- if some node \( n \) is reached from two observables then add to the slice set the transition(s) from \( n \).

Running time: quadratic in number of transitions (excluding time to compute DD, an orthogonal issue).
Finding Least Set Satisfying WCC: Example

1. Add $t_8$ and $t_7$
2. Add $t_2$ and $t_3$
3. Add $t_4$
4. Add $t_1$
Starting with slicing criterion $t_6, t_9$ we
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Starting with slicing criterion \( t_6, t_9 \) we

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Finding Least Set Satisfying WCC: Example

Starting with slicing criterion $t_6$, $t_9$ we

1. add $t_8$ and $t_7$
2. add $t_2$ and $t_3$
Finding Least Set Satisfying WCC: Example

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Finding Least Set Satisfying WCC: Example

Starting with slicing criterion $t_6, t_9$ we

1. add $t_8$ and $t_7$
2. add $t_2$ and $t_3$
3. add $t_4$
Finding Least Set Satisfying WCC: Example

Starting with slicing criterion $t_6, t_9$ we

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Finding Least Set Satisfying WCC: Example

Starting with slicing criterion \( t_6, t_9 \) we

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3. add \( t_4 \)
4. add \( t_1 \)
Starting with slicing criterion $t_6, t_9$ we

1. add $t_8$ and $t_7$
2. add $t_2$ and $t_3$
3. add $t_4$
4. add $t_1$
Finding Least Set Satisfying SCC: Theory

Observe: if WCC holds but SCC does not then we have

\[ t_1 \]

\[ n \]

\[ n_1 \]

\[ n_2 \]

and \( t_1 \) will belong to any superset satisfying SCC.

For a given slicing criterion, there thus exists a least superset that satisfies SCC (and is closed under DD) and we can write an algorithm to iteratively find this set:

- from the observables, do a \textbf{backwards} breadth-first search through transitions not in slice set.
- if some node \( n \) is reached from \textbf{two} observables, or may avoid its observable, then add transition(s) towards observable from \( n \).

Running time: \textbf{quadratic} in number of transitions (excluding time to compute DD, including time to precompute which nodes may avoid which nodes).
Finding Least Set Satisfying SCC: Example

Start with slicing criterion $t_9$, we add:

1. $t_2$
2. $t_8$
3. $t_3$
4. $t_7$

The diagram shows the relationships between states and transitions, with arrows indicating the direction of the transitions.
Finding Least Set Satisfying SCC: Example

Starting with slicing criterion \( t_9 \) we
Finding Least Set Satisfying SCC: Example

Starting with slicing criterion $t_9$ we

1. add $t_2$
Starting with slicing criterion \( t_9 \) we

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Finding Least Set Satisfying SCC: Example

Starting with slicing criterion $t_9$ we

1. add $t_2$
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Finding Least Set Satisfying SCC: Example

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Finding Least Set Satisfying SCC: Example

Starting with slicing criterion $t_9$ we

1. add $t_2$
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4. add $t_7$
Future Work

We would like to see if our ideas could be extended to handle **concurrent programs**, taking inspiration from [Hatcliff et al, SAS 1999] which

- considers **multi-threading** with synchronization through monitors
- defines **various dependencies**, not just data and control but also *divergence, interference, synchronization, ready*
- proposes **bisimulation** as correctness property but does **not** work out the details