The Validity and Soundness of Arguments

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Introduction

Atomic Sentences (summary)

Logical Consequence

Demonstrating Non-consequence
Two main aims of book (p.2):

1. help you learn language of first-order logic (FOL)
2. help you learn notion of logical consequence
Road Map

Two main aims of book (p.2):

1. help you learn language of first-order logic (FOL)
2. help you learn notion of logical consequence

- Chapter 1 takes the first step of (1)
- Chapter 2 takes the first step of (2)
Atomic Sentences

A term $t$ is built from constants and function symbols:

$$\text{father(father(max))}$$

An atomic sentence is a predicate applied to some terms:

$$\text{Older(father(max),max)}$$
Atomic Sentences

A term \( t \) is built from constants and function symbols:

\[
father(father(max))
\]

An atomic sentence is a predicate applied to some terms:

\[
Older(father(max), max)
\]

<table>
<thead>
<tr>
<th>result is</th>
<th>functions</th>
<th>predicates</th>
</tr>
</thead>
<tbody>
<tr>
<td>object</td>
<td>truth value</td>
<td></td>
</tr>
<tr>
<td>lower case</td>
<td>capitalized</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>spelling is</th>
<th>can be nested?</th>
</tr>
</thead>
<tbody>
<tr>
<td>lower case</td>
<td>yes</td>
</tr>
<tr>
<td>capitalized</td>
<td>no</td>
</tr>
</tbody>
</table>
## Example Worlds

<table>
<thead>
<tr>
<th></th>
<th>constants</th>
<th>functions (arity 1)</th>
<th>functions (arity 2)</th>
<th>predicates (arity 1)</th>
<th>predicates (arity 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic</td>
<td>0, 1, 2, ...</td>
<td>sin, cos, +,-</td>
<td></td>
<td></td>
<td>&lt;</td>
</tr>
<tr>
<td>Family</td>
<td>max, claire</td>
<td>father</td>
<td></td>
<td>Pet</td>
<td>Older</td>
</tr>
<tr>
<td>Tarski’s World</td>
<td>a, b, ...</td>
<td></td>
<td>Cube</td>
<td>LeftOf</td>
<td></td>
</tr>
</tbody>
</table>

- many functions and predicates with arity 2 are written infix: $x + y$, $x < y$, $x = y$
- functions can be added to Tarski’s world (p.33, and homework exercises 1.13 & 1.14)
- The identity predicate “=” is relevant in all worlds!
Motivation

An argument is not two persons arguing back and forth, but one person presenting a series of statements in which one, the conclusion, is meant to be a consequence of the others, called the premises.
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Premises

<table>
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<th>a is larger than b</th>
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<tbody>
<tr>
<td>b is larger than c</td>
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</table>

Conclusion

| a is larger than c |
Motivation

An argument is **not** two persons arguing back and forth, but

*one person presenting a series of statements in which one, the **conclusion**, is meant to be a consequence of the others, called the **premises**.*

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| Conclusion | a is larger than c |

**Fitch format**

```
  a is larger than b
  b is larger than c
  a is larger than c
```
Valid and Sound Arguments

Socrates is a man
All men are mortal
Socrates is mortal
Valid and Sound Arguments

Socrates is a man
All men are mortal
Socrates is mortal

This classical argument is

▶ valid: it is not possible for the conclusion to be false if the premises are true.
Valid and Sound Arguments

<table>
<thead>
<tr>
<th>Socrates is a man</th>
<th>true (history)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All men are mortal</td>
<td>true (biology)</td>
</tr>
<tr>
<td>Socrates is mortal</td>
<td></td>
</tr>
</tbody>
</table>

This classical argument is

- **valid**: it is not possible for the conclusion to be false if the premises are true.
- **sound**: it is valid, and its premises are true.
Valid and Sound Arguments

- Socrates is a man: true (history)
- All men are mortal: true (biology)
- Socrates is mortal: true (history: hemlock, 399 BC)

This classical argument is

- **valid**: it is not possible for the conclusion to be false if the premises are true.
- **sound**: it is valid, and its premises are true.
  (so also its conclusion is true)
Unsound Arguments

Scruffy is a man
All men are mortal
Scruffy is mortal

This argument is

- **valid**, as same structure as the previous argument
Unsound Arguments

<p>| | |</p>
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<thead>
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<tr>
<td>Scruffy is a man</td>
<td>false</td>
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<td>All men are mortal</td>
<td>true</td>
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<td>Scruffy is mortal</td>
<td>true</td>
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This argument is

- **valid**, as same structure as the previous argument
- **unsound**, since Scruffy is a cat
  (conclusion happens to be true)
Unsound Arguments

Scruffy is a man  false
All men are mortal  true
Scruffy is mortal  true

This argument is

- **valid**, as same structure as the previous argument
- **unsound**, since Scruffy is a cat
  (conclusion happens to be true)

Red Sox win the World Series each year
Red Sox will win the 2004 World Series
Unsound Arguments

Scruffy is a man \hspace{1em} \text{false}
All men are mortal \hspace{1em} \text{true}
Scruffy is mortal \hspace{1em} \text{true}

This argument is

\begin{itemize}
  \item \textbf{valid}, as same structure as the previous argument
  \item \textbf{unsound}, since Scruffy is a cat
     \hspace{1em} (conclusion happens to be true)
\end{itemize}

Red Sox win the World Series each year \hspace{1em} \text{false}
Red Sox will win the 2004 World Series \hspace{1em} ?

This is also \textbf{valid}, but, alas, \textbf{not sound}.
Invalid Arguments

Socrates is mortal
All men are mortal
Socrates is a man

This argument has a different structure than what we have seen, and is invalid.
## Invalid Arguments

<table>
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This argument has a **different** structure than what we have seen, and is **invalid**.

**Counterexample:** Socrates **might** be a dog
Invalid Arguments

Socrates is mortal
All men are mortal
Socrates is a man

This argument has a **different** structure than what we have seen, and is **invalid**.

**Counterexample:** Socrates **might** be a dog

To decide whether an argument is

- **valid:** it is sufficient to examine the **structure** of the argument
- **sound:** we must examine history, biology, baseball, etc.

Therefore the focus of logic, and this course, is on **validity** of argument, rather than on **soundness**.
Counterexamples (Section 2.5)

Given a purported argument, a **counterexample** is

- a world where the premises are **true** but the conclusion is **false**
- enough to show that the argument is **invalid**: the conclusion does **not** follow from the premises (is **non sequitur**).
Given a purported argument, a **counterexample** is

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Mrs. Smith was stabbed in her bedroom
All doors and windows were locked
Only Mr. Smith and the butler were in the house
Mr. Smith stabbed his wife
Given a purported argument, a **counterexample** is

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- enough to show that the argument is **invalid**: the conclusion does *not* follow from the premises (is **non sequitur**).

Mrs. Smith was stabbed in her bedroom
All doors and windows were locked
Only Mr. Smith and the butler were in the house
Mr. Smith stabbed his wife    the butler did it
Submitting Counterexamples

In homeworks, you’ll often be given an argument and asked to submit a world that serves as a counterexample.

\begin{align*}
\text{LeftOf}(a,b) \\
\text{SameSize}(b,c) \\
\text{LeftOf}(a,c)
\end{align*}
Submitting Counterexamples

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\[
\begin{align*}
\text{LeftOf}(a,b) \quad & \text{SameSize}(b,c) \quad \text{LeftOf}(a,c) \\
\text{Counterexample: a world with only small cubes, arranged like} \\
& \begin{array}{ccc}
& c & & a & b \\
\end{array}
\end{align*}
\]
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**Counterexample:** a world with only small cubes, arranged like\[c\quad a \quad b\]

\[
\begin{align*}
\text{LeftOf}(a,b) \\
b = c \\
\text{LeftOf}(a,c)
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**Counterexample:**
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\textbf{Counterexample: } a \text{ world with only small cubes, arranged like } \begin{array}{ccc} c & a & b \end{array}

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\textbf{Counterexample: } \text{ none, as argument is valid}