CIS 301, Spring 2008, Exam I, model solutions

Question 1

A	B	C	$P:\ A \leftrightarrow (B \wedge \neg C)$	$Q:\ \neg((\neg B \vee C) \rightarrow A)$
T	T	T	**F** F F	**F** F T T
T	T	F	**T** T T	**F** F F T
T	F	T	**F** F F	**F** T T T
T	F	F	**F** F T	**F** T T T
F	T	T	**T** F F	**T** F T F
F	T	F	**F** T T	**F** F F T
F	F	T	**T** F F	**T** T T F
F	F	F	**T** F T	**T** T T F

P is true in every row where Q is true, showing that P is a tautological consequence of Q. The completeness result mentioned on p.219 now tells us that P can be proved from Q.

On the other hand, line 2 shows that Q is not a tautological consequence of P. From the soundness result mentioned on p.215, we infer that Q cannot be proved from P.

Question 2

1. $(P \rightarrow \neg R) \vee (Q \rightarrow \neg R)$	We write line numbers even though not required
│ 2. R	"Assume R"
│ │ 3. $P \wedge Q$	Starting proof by contradiction
│ │ 4. P	$\wedge$ **Elim** (3)
│ │ │ 5. $P \rightarrow \neg R$	Case 1
│ │ │ 6. $\neg R$	$\rightarrow$ **Elim** (4, 5)
│ │ │ 7. $\perp$	$\perp$ **Intro** (2, 6)
│ │ 8. Q	$\wedge$ **Elim** (3)
│ │ │ 9. $Q \rightarrow \neg R$	Case 2
│ │ │ 10. $\neg R$	$\rightarrow$ **Elim** (8, 9)
│ │ │ 11. $\perp$	$\perp$ **Intro** (2, 10)
│ │ 12. $\perp$	$\vee$ **Elim** (1, 5–7, 9–11)
│ 13. $\neg(P \wedge Q)$	$\neg$ **Intro** (3–12)
14. $R \rightarrow \neg(P \wedge Q)$	$\rightarrow$ **Intro** (2–13)

Question 3

First note that we have the following *modularity* property: changes to the preamble can never violate proof item 2 or proof item 3, while changes to the loop body can never violate proof item 1 or proof item 3.

Change 2 violates item 1, since $0 \neq \mathrm{fac}(0)$. (But not item 4, since that argument does not depend on $\mathtt{y}$.) That is, the invariant is never established (and in fact, the program is wrong as it always returns $\mathtt{y} = 0$).

Change 3 violates item 4, since if $\mathtt{x} = 0$, the identifier $\mathtt{z}$ will never equal $\mathtt{x}$. (But item 1 is not violated, as $1 = \mathrm{fac}(1)$.)

Change 4 violates item 2, since we now have $\mathtt{y}' = \mathtt{y} \cdot \mathtt{z} = \mathrm{fac}(\mathtt{z}) \cdot \mathtt{z}$ which is not equal to $\mathrm{fac}(\mathtt{z}') = \mathrm{fac}(\mathtt{z} + 1) = \mathrm{fac}(\mathtt{z}) \cdot (\mathtt{z} + 1)$. (In particular, after the first loop iteration we have $\mathtt{y}' = 0 \neq 1 = \mathrm{fac}(1) = \mathrm{fac}(\mathtt{z}')$.)