CIS 301, Spring 2008, Exam II, model solutions

Question 1
∀x(Tet(x) → ¬Small(x))
∃x(Large(x) ∧ Dodec(x) ∧ ∀y((Large(y) ∧ Dodec(y)) → y = x))
∀x(Cube(x) → ∃y(y ≠ x ∧ SameSize(x, y)))

Question 2
{0 = 4 · (x − x)}
    y := 0;
{y = 4 · (x − x)} Assignment
    w := x;
{y = 4 · (x − w)} Assignment
        while w ≠ 0 do
            {y + 4 = 4 · (x − (w − 1))} Implies
                y := y + 4;
            {y = 4 · (x − (w − 1))} Assignment
                w := w − 1
            {y = 4 · (x − w)} Assignment
            od
{y = 4 · (x − w) ∧ w = 0} WhileFalse
{y = 4 · x} Implies

Since 0 = 4 · (x − x) always holds, φ₀ could be anything. On the other hand, remember that the above proof shows only partial correctness. To ensure termination, we must demand that x ≥ 0.

Question 3
1. ∀x(P(x) → (Q(x) ∨ R(x)))
2. ∃y(P(y) ∧ ¬Q(y))
3. ∀a P(a) ∧ ¬Q(a) (name the object satisfying P but not Q)
4. P(a) → (Q(a) ∨ R(a)) ∀ Elim : 1
5. P(a) ∧ R(a) TautCon : 3, 4
6. ∃w(P(w) ∧ R(w)) ∃ Intro : 5
7. ∃w(P(w) ∧ R(w)) ∃ Elim : 2, 3–6

Question 4
If we replace SameRow and SameCol by SameSize then (a) says that all cubes have the same size — which clearly does not imply (b). To make (b) a FO consequence of (a), we must capture that two different objects cannot occupy the same square; this can be achieved by the axiom

∀x∀y((SameRow(x, y) ∧ SameCol(x, y)) → x = y).

If we replace SameRow and SameCol by Larger then (a) says that all cubes are larger than each other, which can only hold if there are no cubes — a stronger property than (b), and hence not implied by (b). To make (a) a FO consequence of (b), we must capture that a cube is in the same row and same column as itself; this can be achieved by the axiom (where of course we could omit the antecedent “Cube(x)”) 

∀x(Cube(x) → (SameRow(x, x) ∧ SameCol(x, x))).