The Validity and Soundness of Arguments

Torben Amtoft
Kansas State University
Introduction

Atomic Sentences (summary)

Logical Consequence

Demonstrating Non-consequence
Two main aims of book (p.2):

1. help you learn language of first-order logic (FOL)
2. help you learn notion of logical consequence
Two main aims of book (p.2):

1. help you learn language of first-order logic (FOL)
2. help you learn notion of logical consequence

- Chapter 1 takes the first step of (1)
- Chapter 2 takes the first step of (2)
Atomic Sentences

A term $t$ is built from constants and function symbols:

$$\text{father}(\text{father}(\text{max}))$$

An atomic sentence is a predicate applied to some terms:

$$\text{Older}(\text{father}(\text{max}), \text{max})$$
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An atomic sentence is a predicate applied to some terms:
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<table>
<thead>
<tr>
<th></th>
<th>functions</th>
<th>predicates</th>
</tr>
</thead>
<tbody>
<tr>
<td>result is</td>
<td>object</td>
<td>truth value</td>
</tr>
<tr>
<td>spelling is</td>
<td>lower case</td>
<td>capitalized</td>
</tr>
<tr>
<td>can be nested?</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>
## Example Worlds

<table>
<thead>
<tr>
<th></th>
<th>constants</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>arity 1</td>
<td>arity 2</td>
</tr>
<tr>
<td>Arithmetic</td>
<td>0, 1, 2, ...</td>
<td>sin, cos</td>
<td>+,-</td>
</tr>
<tr>
<td>Family</td>
<td>max, claire</td>
<td>father</td>
<td></td>
</tr>
<tr>
<td>Tarski’s World</td>
<td>a, b, ...</td>
<td>Cube</td>
<td>LeftOf</td>
</tr>
</tbody>
</table>

- Many functions and predicates with arity 2 are written infix: 
  \[ x + y, x < y, x = y \]
- Functions can be added to Tarski’s world
  (p.33, and homework exercises 1.13 & 1.14)
- The identity predicate "=" is relevant in all worlds!
Motivation

An argument is not two persons arguing back and forth, but one person presenting a series of statements in which one, the conclusion, is meant to be a consequence of the others, called the premises.
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<tr>
<th>Premises</th>
<th>Conclusion</th>
</tr>
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<tbody>
<tr>
<td>a is larger than b</td>
<td>a is larger than c</td>
</tr>
<tr>
<td>b is larger than c</td>
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Torben Amtoft  Kansas State University  The Validity and Soundness of Arguments
Motivation

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Premises

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Conclusion

| a is larger than c |

Fitch format

```
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Valid and Sound Arguments

Socrates is a man
All men are mortal
Socrates is mortal

This classical argument is ▶ valid: it is not possible for the conclusion to be false if the premises are true.
▶ sound: it is valid, and its premises are true (so also its conclusion is true).
Valid and Sound Arguments

- Socrates is a man
- All men are mortal
- Socrates is mortal

This classical argument is

- **valid**: it is not possible for the conclusion to be false if the premises are true.
Socrates is a man \(\text{true}\) (history)
All men are mortal \(\text{true}\) (biology)
Socrates is mortal

This classical argument is

- **valid**: it is not possible for the conclusion to be false if the premises are true.
- **sound**: it is valid, and its premises are true.
Valid and Sound Arguments

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<th>Socrates is a man</th>
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<td>All men are mortal</td>
<td>true (biology)</td>
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<tr>
<td>Socrates is mortal</td>
<td>true (history: hemlock, 399 BC)</td>
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This classical argument is

- **valid**: it is not possible for the conclusion to be false if the premises are true.
- **sound**: it is valid, and its premises are true.
  (so also its conclusion is true)
Unsound Arguments

- Scruffy is a man
- All men are mortal
- Scruffy is mortal

This argument is

- **valid**, as same structure as the previous argument
Unsound Arguments

<table>
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<tr>
<th>Scruffy is a man</th>
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This argument is

- **valid**, as same structure as the previous argument
- **unsound**, since Scruffy is a cat
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| Red Sox will win the 2004 World Series |
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<tr>
<td>Red Sox will win the 2004 World Series</td>
<td>?</td>
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This is also **valid**, but, alas, **not sound**.
Invalid Arguments

Socrates is mortal
All men are mortal
Socrates is a man

This argument has a different structure than what we have seen, and is invalid.
Invalid Arguments

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**Counterexample:** Socrates might be a dog
Invalid Arguments

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Counterexample: Socrates might be a dog

To decide whether an argument is

- valid: it is sufficient to examine the structure of the argument
- sound: we must examine history, biology, baseball, etc.

Therefore the focus of logic, and this course, is on validity of argument, rather than on soundness.
Counterexamples (Section 2.5)

Given a purported argument, a **counterexample** is

- a world where the premises are **true** but the conclusion is **false**
- enough to show that the argument is **invalid**: the conclusion does **not** follow from the premises (is **non sequitur**).
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Mrs. Smith was stabbed in her bedroom
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Only Mr. Smith and the butler were in the house
Mr. Smith stabbed his wife
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Atta was in Virginia on April 4, 2001
Atta was in Florida on April 11, 2001
Atta cannot have met with Iraqi agents in Prague on April 8, 2001
Atta took an airplane...
Submitting Counterexamples

In homeworks, you’ll often be given an argument and asked to submit a world that serves as a counterexample.

\[
\begin{align*}
\text{LeftOf}(a,b) \\
\text{SameSize}(b,c) \\
\text{LeftOf}(a,c)
\end{align*}
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**Counterexample:** a world with only small cubes, arranged like 

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   c   a   b
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**Counterexample:** a world with only small cubes, arranged like 
\[
\begin{array}{ccc}
  & c & \\
  a & & b \\
\end{array}
\]

\[
\begin{align*}
&\text{LeftOf}(a,b) \\
&b = c \\
&\text{LeftOf}(a,c)
\end{align*}
\]

**Counterexample:** none, as argument is valid