A brief introduction to
static program analysis

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What is static program analysis?

It is the extraction of a program's properties in advance of the program's execution.

Example properties:

- the program will not generate a run-time exception (error)
- the program will generate an output that has a desirable property
- the program's internal statements have desirable properties that admit optimization

Standard techniques:

- type checking
- iterative dataflow analysis
- theorem provimg

An example Python program

Let n be some input integer:

```
p0 : i = n;  x = 0;
p1 : while i != 0 :
       p2 : x = x + 1;  i = i - 1
p3 : print x
```

What properties can we extract?

- the program will not generate a type-mismatch exception
- the definitions (assignments) at point p_0 possibly reach p_3
- the program satisfies the postcondition, $x = n$

Type checking the Python program

p_0: `i = n;  x = 0;`
p_1: `while i != 0 :`
    p_2: `x = x + 1;  i = i - 1`
p_3: `print x`

The program is well-typed: it won't generate a mismatch exception.

$$\Gamma \vdash 0 : \mathrm{int} \qquad \frac{\Gamma \vdash e_1 : \mathrm{int} \quad \Gamma \vdash e_2 : \mathrm{int}}{\Gamma \vdash e_1 + e_2 : \mathrm{int}} \qquad \frac{\Gamma \vdash e_1 : \mathrm{int} \quad \Gamma \vdash e_2 : \mathrm{int}}{\Gamma \vdash e_1 == e_2 : \mathrm{bool}}$$

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash x = e : \Gamma + [x \mapsto \tau]} \qquad \frac{\Gamma \vdash c_1 : \Gamma_1 \quad \Gamma_1 \vdash c_2 : \Gamma_2}{\Gamma \vdash c_1 ; c_2 : \Gamma_2} \qquad \frac{\Gamma \vdash e : \mathrm{bool} \quad \Gamma \vdash c : \Gamma}{\Gamma \vdash \mathit{while}\ e \colon c : \Gamma}$$

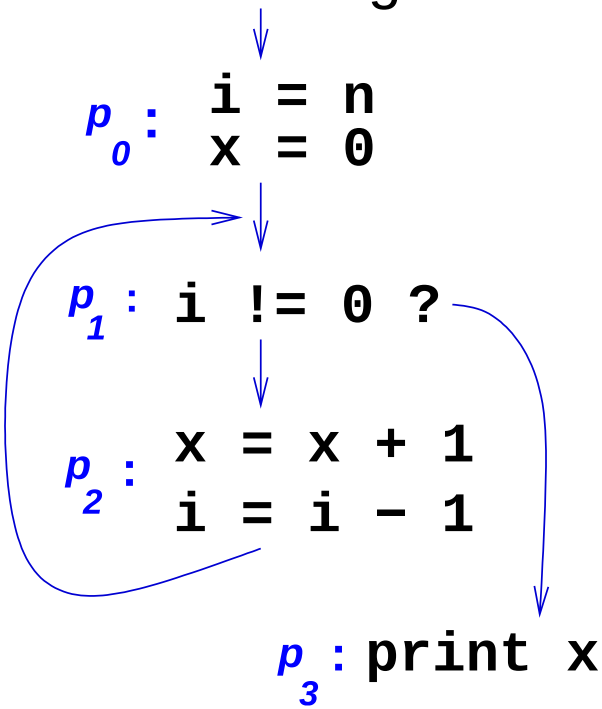
Although it's drawn as an (inverted) deduction, type checking is implemented as a traversal of the program's parse tree:

$$\frac{[\,] \vdash \texttt{i = n} \ [i \mapsto \mathrm{int}] \qquad [i \mapsto \mathrm{int}] \vdash \texttt{x = 0} : \Gamma_0 \qquad \dfrac{\Gamma_0 \vdash \texttt{i != n} : \Gamma_0 \qquad \dfrac{\Gamma_0 \vdash \texttt{x = x + 1} : \Gamma_0 \qquad \Gamma_0 \vdash \texttt{i = i - 1} : \Gamma_0}{\Gamma_0 \vdash \texttt{x = x + 1; i = i - 1} : \Gamma_0}}{\Gamma_0 \vdash \texttt{while i != 0: x = x + 1; i = i - 1} : \Gamma_0}}{[\,] \vdash \texttt{i = n; x = 0; while i != 0: x = x + 1; i = i - 1} : \Gamma_0}$$

Let $\Gamma_0 = [\, x \mapsto \mathrm{int},\ i \mapsto \mathrm{int}]$

Reaching definitions calculated by dataflow analysis

Does the assignment at p_i reach point p_j?



$$\mathrm{in}_{p_i} = \bigcup_{p' \in \mathrm{pred}\, p_i} \mathrm{out}_{p'}$$

$$\mathrm{out}_{p_i} = \mathrm{in}_{p_i} - \{p_x | p_x \equiv x = e\} \cup \{p_i\},$$

for $p_i \equiv x' = e'$

$$\mathrm{out}_{p_i} = \mathrm{in}_{p_i}, \quad \text{for } p_i \equiv e?$$

For the example program, the equations for reaching definitions are solved iteratively as

$$\mathrm{in}_{p_0} = \{\} \qquad \mathrm{in}_{p_1} = \{p_0, p_2\}$$

$$\mathrm{in}_{p_2} = \{p_0, p_2\} \quad \mathrm{in}_{p_3} = \{p_0, p_2\}$$

Partial correctness proved within Hoare logic

```
p0 : i = n;  x = 0;
p1 : while i != 0 :
        p2 : x = x + 1;  i = i - 1
p3 : print x
```

$$\{[e/x]P\}\ x = e\ \{P\}$$

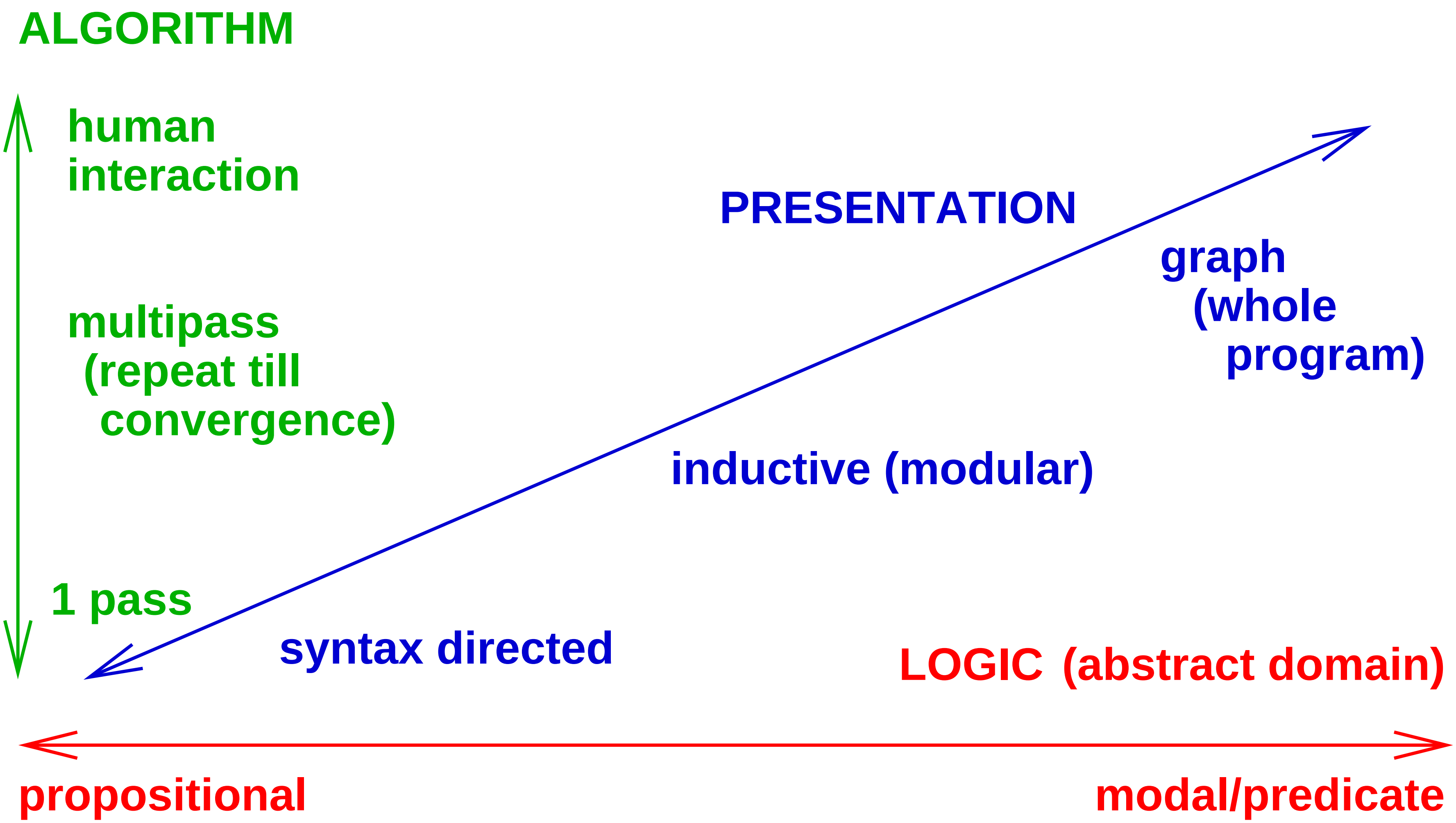
$$\frac{\{P\}\ c1\ \{Q\}\ c2\ \{R\}}{\{P\}\ c1; c2\ \{R\}}$$

$$\frac{\{e \wedge P\}\ c\ \{P\}}{\{P\}\ \textit{while}\ e :\ c\ \{\neg e \wedge P\}}$$

$$\frac{\dfrac{\dfrac{\{x+1=n-i+1\}\ \texttt{x = x + 1}\ \{x=n-i+1\}\ \texttt{i = i - 1}\ \{x=n-i\}}{\{i \mathrel{!}= 0\ \&\ x=n-i\}\ \texttt{x = x + 1; i = i - 1}\ \{x=n-i\}}}{\{x=n-i\}\ \texttt{while i != 0: x = x + 1; i = i - 1}\ \{x=n\}}}{\{true\}\ \texttt{i = n; x = 0}\ \{x=n-i\}\ \texttt{while i != 0: x = x + 1; i = i - 1}\{x=n\}}$$

One must discover the loop invariant, $x = n - i$, to accomplish the proof.

Three axes of static analyses



Some standard static analyses

	logic	algorithm	presentation
type checking	propositional logic: $\mathtt{int}$, $\mathtt{int} \to \mathtt{bool}$	one pass (traverse syntax tree)	syntax-directed
ML type inference	shallow $\forall$ logic: $\forall \alpha . \alpha \to \alpha$	one pass + unification	syntax-directed
a.i.-based dataflow analysis	propositional logic (token sets)	iterate until convergence	graph-based
model checking	LTL, ACTL (modal-like logics)	iterate forever (!)	graph-based
theorem proving, LF	predicate logics	human interaction	inductive, stated axiomatically

A "hybrid" analysis: predicate abstraction

We wish to prove that $\mathtt{z} \geq \mathtt{x} \wedge \mathtt{z} \geq \mathtt{y}$ at p_3:

***p*₀** **:** `if x < y`
***p*₁** **:** `  then z = y`
***p*₂** **:** `  else z = x`
p*₃** **:** ***exit

$$\begin{tikzcd}[column sep=small]
 & p_0, \langle ?, ?, ? \rangle \arrow[dl] \arrow[dr] & \\
p_1, \langle t, ?, ? \rangle \arrow[d] & & p_2, \langle f, ?, ? \rangle \arrow[d] \\
p_3, \langle t, t, t \rangle & & p_3, \langle f, t, t \rangle
\end{tikzcd}$$

We choose three predicates,

$$\phi_1 = \mathtt{x} < \mathtt{y}$$

$$\phi_2 = \mathtt{z} \geq \mathtt{x}$$

$$\phi_2 = \mathtt{z} \geq \mathtt{y}$$

and compute their values at the program's points. The predicates' values come from the domain, $\{t, f, ?\}$. (Read $?$ as $t \vee f$.)

At all occurrences of p_3 in the abstract trace, $\phi_2 \wedge \phi_3$ holds.

When a goal is undecided, refinement is necessary

Prove $\phi_0 \equiv \mathtt{x} \geq \mathtt{y}$ at p_4:

```
p0 : if !(x >= y)
p1 : then { i = x;
       p2 :  x = y;
       p3 :  y = i;
p4 : }
```

$$\begin{array}{ccc} & p_0, \langle ? \rangle & \\ \swarrow & & \searrow \\ p_1, \langle f \rangle & & p_4, \langle t \rangle \\ p_2, \langle f \rangle & & \\ p_3, \langle t \rangle & & \\ p_4, \langle ? \rangle & & \end{array}$$

To decide the goal, we must refine the state by adding a needed auxiliary predicate: $wp(\mathtt{y} = \mathtt{i}, \mathtt{x} \geq \mathtt{y}) = (\mathtt{x} \geq \mathtt{i}) \equiv \phi_1$.

$$\begin{array}{ccc} & p_0, \langle ?, ? \rangle & \\ \swarrow & & \searrow \\ p_1, \langle f, ? \rangle & & p_4, \langle t \rangle \\ p_2, \langle f, t \rangle & & \\ p_3, \langle t, t \rangle & & \\ p_4, \langle t, t \rangle & & \end{array}$$

because $\mathtt{x} \not\geq \mathtt{y}$ and $\mathtt{x} \geq \mathtt{i}$ imply $\mathtt{y} > \mathtt{i}$ implies $\mathtt{x}_{new} \geq \mathtt{i}$

But incremental predicate refinement cannot synthesize many interesting loop invariants. For this example:

p_0 : `i = n; x = 0;`
p_1 : `while  i != 0  {`
  p_2 : `x = x + 1;  i = i - 1;`
  `}`
p_3 : `goal: x = n`

We find that the initial predicate set, $P_0 \equiv \{i = 0, x = n\}$, does not validate the loop body.

The first refinement suggests we add $P_1 \equiv \{i = 1, x = n - 1\}$ to the program state, but this fails to validate a loop that iterates more than once.

Refinement stage j adds predicates $P_j \equiv \{i = j, x = n - j\}$; the refinement process continues forever!

The loop invariant is $x = n - i$:-)

Mr. Gedell will present interesting combinations and variations of the standard analyses....

References This talk: www.cis.ksu.edu/~schmidt/presentations

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