Here is Milner's original definition of simulation on trees:

"T simulates U" written $T \precsim U$ iff $\forall i \geq 0 \; T \precsim_i U$,

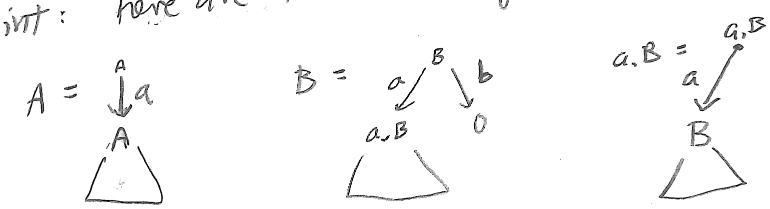
where
$$\begin{cases} X \precsim_0 Y & \text{for all trees, } X, Y \\ X \precsim_{i+1} Y & \text{iff for all actions, } m, \; Y \xrightarrow{m} Y' \text{ implies } X \xrightarrow{m} X' \text{ and } X' \precsim_i Y' \end{cases}$$

Question 1:

Use this definition to prove

$B \precsim A$, where
$$A = a.A$$
$$B = a.a.B + b.0$$
halt

Hint: here are the trees you will examine:



(When you finish, the proof you make of the inductive step will be exactly the proof you make in a David-Park-style proof by coinduction.)

Park's co-induction principle (in two stages):

A recursive predicate, $P(t) = \cdots t \cdots P(t') \cdots$ defines a set, PTree, of trees that have P:

$$PTree = \bigcap_{i \geq 0} PT_i \quad \begin{cases} PT_0 = \text{all trees} \\ PT_{i+1} = \{ t \mid \cdots t \cdots \ t' \in PT_i \cdots \} \end{cases}$$

Example: $noSoda(t) \triangleq t \xrightarrow{m} t',\ m \neq soda,\ noSoda(t')$.

This defines the set, $no\$Tree = \bigcap_{i \geq 0} no\$_i$, where $\begin{cases} no\$_0 = \text{all trees} \\ no\$_{i+1} = \{ t \mid t \xrightarrow{m} t',\ m \neq soda,\ no\$_i(t') \} \end{cases}$

Thus, $noSoda(t_0)$ holds true
<u>iff</u> $t_0 \in no\$Tree$.
<u>iff</u> $t_0 \in no\$_i$, for all $i \geq 0$.

Question 2A:

for tree $M = c.f.M$, prove $noSoda(M)$ holds true.

(Hint: you must also prove at the same time that $noSoda(f.M)$)

The co-induction proof rule:

we have a recursive property, $P(t) = \ldots t \ldots P(t') \ldots$,

which implicitly defines a set, PTree $\subseteq$ Tree,

Say we have tree t_0, and we want to prove $P(t_0)$:

(i) define a set, myP $\subseteq$ Tree, such that $t_0 \in$ myP

(ii) prove myP $\subseteq \{ t \mid \ldots t \ldots t' \in$ myP $\ldots \}$

i.e., prove, for each $t'' \in$ myP, that

$\ldots t'' \ldots t''' \in$ myP $\ldots$ holds true.

Question 2B: Again, for

$noSoda(t) = t \xrightarrow{m} t'$, $m \neq soda$, $noSoda(t')$

Prove $noSoda(M)$, where $M = c.f.M$

Hint: define myNS = { ______ , ______ }

prove, for each $t'' \in$ myNS,

that $t'' \xrightarrow{m} t'''$, $m \neq soda$, $t''' \in$ myNS

(Final question: can you prove $noSoda(N)$, where $N = c.(f.N + g.0)$?

Why not?)

How about $ifSoda(N)$, where $ifSoda(t) =$ if $t \xrightarrow{m} t'$ then $m \neq soda$, $ifSoda(t')$?