Multitape Turing Machines

- $\delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R\}^k$
- IDs are $(k + 1)$-tuples: $Q \times (\Gamma^* \# \Gamma^*)^k$

- $\vdash$ is defined in the natural way

- $L(M)$ is the set of all strings $w$ such that

  \[
  (q_0, \# w, \# , \ldots, \#) \vdash^* (q, \alpha_1 \# \beta_1, \ldots, \alpha_k \# \beta_k)\]

  for some $q \in F$, $\alpha_1, \ldots, \alpha_k, \beta_1, \ldots, \beta_k \in \Gamma^*$.

To simulate a $k$-tape TM with a TM:

- Use a $k$-track TM
- Record the state in finite control
- Record each $\alpha \# \beta$ on a separate track
- To simulate a single transition:
  - assume the head is to the left of every $\#$
  - scan right, recording in finite control the symbol following each $\#$ until all $k$ symbols have been recorded
  - scan left, updating each track
Nondeterministic TMs

- $\delta : Q \times \Gamma \rightarrow 2^{Q \times \Gamma \times \{L,R\}}$

To simulate an NTM with a 2-tape TM:

- Do a breadth-first search of all computations
- Use one tape as a FIFO queue containing IDs beginning and ending with *
- $\delta$ is stored in finite control

```plaintext
while the queue is not empty do
    find state $q$ and current symbol $X$ in first ID
    for each $(p, Y, D) \in \delta(q, X)$ do
        if $p \in F$ then accept
        copy first ID from queue to second tape, applying transition $(q, X) \rightarrow (p, Y, D)$
        move ID from second tape to end of queue
        overwrite first ID in the queue with blanks
```

3

4
**Claim:** Every CFL is recursive.

**Proof sketch:** Let $L$ be a CFL.

- Let $G$ be a CNF grammar such that $L(G) = L - \{e\}$.
- We will construct a 2-track NTM $M$ such that $L(M) = L(G)$, and $M$ is guaranteed to halt.
- It is easy to modify $M$ to accept $\epsilon$ if $\epsilon \in L$.
- $M$ will nondeterministically find a derivation of input $w$.

write $S$ on track 2, move right

**while** current symbol on track 1 is nonblank **do**

move left

**while** the current symbol is nonblank **do**

nondeterministically move left or break

nondeterministically select production $A \rightarrow \alpha$, where $A$ is current symbol on track 2

apply $A \rightarrow \alpha$ to track 2

leaving tape head to the right of last nonblank on track 2

move left
while the current symbol is nonblank do
verify that track symbols match or
that there is a production \( A \rightarrow a \)
where \( a \) is on track 1, \( A \) on track 2
move left

- \( M \) accepts \( w \) iff \( w \in L(G) \).
- Each iteration of the first loop either
  increases the number of nonblank symbols
  on track 2 or increases the number of
  terminals.
- The first loop must eventually terminate.
- Therefore, \( M \) always terminates.
Restricted Turing Machines

- Semi-infinite tapes
  - We can simulate a TM tape with a semi-infinite tape by “folding over”.

- Multistack TMs
  - We can simulate a TM tape with two stacks
  - The TM tape is “split” at the tape head position

- Counter machines