Relationships between language classes

Claim: If \( L \) is recursive, then \( L \) is RE.

Claim: Let \( L = \{ww \mid w \in \{0, 1\}^*\} \). Then \( L \) is recursive.

Claim: There is an RE language \( L \subseteq \{0, 1\}^* \) that is not recursive.

For \( |\Sigma| \geq 2 \), we have the following:

\[
\text{regular} \subset \text{CFL} \subset \text{recursive} \subset \text{RE}
\]

High-level TM Constructs

- Storage in finite-state control
- Multiple tracks
- Subroutines
- High-level language constructs: if, while
2-track TM to accept \( \{ww \mid w \in \{0, 1\}^* \} \):

**while** symbol on track 2 is \( B \) **do**
- write 0 on track 2, move right
- scan right to \( B \)
- scan left across \( 1^* \) on track 2
- replace \( B \) with 1 on track 2, move left
- scan left across \( B^* \) on track 2, move right

**while** not reading \( B \) **do**
- \( c \leftarrow \) symbol on track 1
- write 0 on track 2, move left
- scan left to \( B \), move right
- **if** symbol on track 1 is not \( c \) **reject**
- write \( B \), move right
- scan right across \( 0^* \) on track 2

**accept**