Ambiguous CFGs

Let $G = (\{S\}, T, P, S)$, where

$$T = \{\text{if, then, else, } b, c\}$$

and $P$ is given by

$$S \rightarrow \text{if } b \text{ then } S \mid \text{if } b \text{ then } S \text{ else } S \mid c$$

Let $w = \text{if } b \text{ then if } b \text{ then } c \text{ else } c$.

We can parse $w$ using $G$ as follows:
We can also parse \( w \) as follows:

\[
S \\
\to \text{if } b \text{ then } S \text{ else } S \\
\to \text{if } b \text{ then } S \ c \\
\to c
\]

**Defn:** A CFG \( G = (V, T, P, S) \) is said to be ambiguous if there are strings \( w \in T^* \), \( \alpha, \beta, \gamma_1, \gamma_2 \in (V \cup T)^* \), and a symbol \( A \in V \), such that

- \( S \xrightarrow{*} \alpha A \beta \xrightarrow{*} \alpha \gamma_1 \beta \xrightarrow{*} w \);
- \( S \xrightarrow{*} \alpha A \beta \xrightarrow{*} \alpha \gamma_2 \beta \xrightarrow{*} w \); and
- \( \gamma_1 \neq \gamma_2 \).

**Defn:** A CFL \( L \) is said to be inherently ambiguous if there there is no unambiguous CFG \( G \) such that \( L(G) = L \).