Exercise Sheet 2
CS 2210 Logic for Computer Scientists (Hitzler)
Solutions due: Tuesday January 27, 2015, 11am

Exercise 6 Let $L = (V, C, R)$ with $V = \{w, y\}$, $C = \{d, e\}$ and $R = \{r, s\}$ where r has arity 1 and s has arity 2. Which of the following are atoms over L? Which are ground atoms? Justify your answers.

(a) $d(w, w)$ (b) $r(d, e)$ (c) $s(w, w)$ (d) $r(y)$

Exercise 7 Let $L = (V, C, R)$ with $V = \{x, y\}$, $C = \{\text{barack}, \text{michelle}, \text{craig}, \text{malia}\}$ and $R = \{\text{motherOf}, \text{parentOf}, \text{grandmotherOf}\}$, all with arity 2.
Which of the Datalog facts (1) to (9) from Example 1.1.1 are atoms over L? Justify your answers.

Exercise 8 Write a Datalog program which captures the following natural language sentences.
(a) If somebody is an orphan, then all his parents are dead.
(b) Every orphan is a human being.
(c) Somebody's father is also that person's parent.
(d) Harry Potter is an orphan.
(e) James Potter is the father of Harry Potter.

Exercise 9 Give three distinct Herbrand interpretations for the following Datalog program, where a, b are constants.

$$q(a)$$
$$p(b)$$
$$q(x) \rightarrow p(x)$$
$$q(y) \wedge p(y) \rightarrow r(b)$$

Exercise 10 Evaluate the following.
(a) $(p(x, y, x) \wedge q(x, y, y) \wedge r(y, y) \rightarrow t(x))[x/a, y/b] = \ldots$
(b) $(p(x) \wedge q(x) \rightarrow r(x))[x/c][x/d] = \ldots$
(c) $(q(a, x) \wedge p(x, y) \wedge q(y, a) \rightarrow r(y))[x/a][x/b] = \ldots$
(d) $(p(x, x) \wedge q(x, y) \rightarrow p(x, y))[y/b][y/c][x/b] = \ldots$

Exercise 11 Which of the substitutions in Exericse 10 are ground substitutions?

Exercise 12 Give the grounding of the Datalog program from Exercise 9.

Exercise 13 Give a Herbrand model for the Datalog program in Exercise 9.

Exercise 14 Give three distinct Herbrand models for the Datalog program P consisting of the following rules.

$$p(a, b)$$
$$q(c)$$
$$p(x, y) \rightarrow q(x)$$