On the Applicability of Weyuker Property 9 to Object-Oriented Structural Inheritance

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Abstract

We formally show that particular classes of inheritance metrics (that include the above proposals) that are defined on a directed graph abstraction of the inheritance structure and that are contrived on the assumptions and definitions given by Chickering and McKeown, can never satisfy Property 9.

Weyuker's Properties

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Nonempty</td>
<td>(8) If R; P = Q, then μP = μQ.</td>
</tr>
<tr>
<td>2</td>
<td>Granularity</td>
<td>Let ε be a non-negative number. Then there are finitely many programs of complexity ε.</td>
</tr>
<tr>
<td>3</td>
<td>Nonuniqueness</td>
<td>There are distinct programs P and Q such that μP = μQ.</td>
</tr>
<tr>
<td>4</td>
<td>Design details matter</td>
<td>(9) If P = Q, then μP = μQ.</td>
</tr>
<tr>
<td>5</td>
<td>Mononicity</td>
<td>(7) If P and Q are both nonempty, then μP ≤ μQ.</td>
</tr>
<tr>
<td>6</td>
<td>Nonequivalence of interaction</td>
<td>(6) If P and Q are both nonempty, then μP ≠ μQ.</td>
</tr>
<tr>
<td>7</td>
<td>Interaction among statements</td>
<td>Not considered for object-oriented metrics (4).</td>
</tr>
<tr>
<td>8</td>
<td>No change on nesting</td>
<td>If P is renaming of Q then μP = μQ.</td>
</tr>
<tr>
<td>9</td>
<td>Interaction CN increase complexity</td>
<td>(5) If P and Q are both nonempty, then μP ≤ μQ.</td>
</tr>
</tbody>
</table>

Grubstake definition

Baker et al. [10] have described a methodology for defining a structural metric which is as follows: First, define an abstraction of the document (the object of measurement), then define an order on the abstraction, and then define an order-preserving map from the abstraction to the real numbers (or any other appropriate number system). We assume that this procedure is observed in the definition of (structural) inheritance metrics that are described subsequently.

Def 1

Definition 1. In an object-oriented system design, let $C = \{c_1, c_2, \ldots, c_n\}$ be the finite set of classes and let $\text{INH}$ be the inheritance relation in $C$ such that $c_i \text{ INH} c_j$, for $c_i, c_j \in C, c_i \neq c_j$. If $c_i$ inherits from $c_j$ and there is no $c_k \in C$ such that $c_k$ inherits from $c_i$ and $c_k$ inherits from $c_j$, i.e., $c_j$ covers $c_i$.

Concatenation

Definition 2. Let $\oplus$ be the operation of concatenation on $G$ such that for vertices $v_i, v_j \in V$, $v_i \oplus v_j$ is the replacement of $v_i$ and $v_j$ by a single new vertex. Each edge that was incident on (from) either $v_i$ or $v_j$ is now incident on (from) the new vertex, provided the edge does not represent a violation of the “cover” property, in which case it is deleted. Only one of resultant parallel edges is retained and self-loops are deleted.
Theorem 1

Theorem 1. Consider the digraph $G = (V,E)$. Let $M^*$ be an inheritance metric and let $v$ be any vertex in $V$. If $M^*(v)$ is a count of the number of elements in a subset $S$ of $V$ containing $v$, then $M^*$ cannot satisfy Property 9 under $\otimes$.

Corollary 1

Corollary 1. Consider the digraph $G = (V,E)$. Let $M^*$ be an inheritance metric and let $v$ be any vertex in $V$. If $M^*(v)$ is a count of the number of elements in a subset $S$ of $V$ defined by $v$ then $M^*$ cannot satisfy Property 9 under $\otimes$.

Theorem 2

Theorem 2. Consider the digraph $G = (V,E)$. Let $M^*$ be an inheritance metric and let $v$ be any vertex in $V$. If $M^*(v)$ is a count of the number of elements in a subset $S$ of $E$ in which there is at least one ordered pair with $v$ as one of the elements, then $M^*$ cannot satisfy Property 9 under $\otimes$.

Corollary 2

Corollary 2. Consider the digraph $G = (V,E)$. Let $M^*$ be an inheritance metric and let $v$ be any vertex in $V$. If $M^*(v)$ is defined as a linear combination of $|S_1|$ and $|S_2|$ where $S_1, S_2 \subseteq E$ in which there is at least one ordered pair with $v$ as one of the elements then $M^*$ cannot satisfy Property 9 under $\otimes$.

Generalization 1

Since a digraph is a suitable abstraction for any document that involves discrete elements and their relationships, the discussion in the previous section can serve as a pointer to the fact that any metric that satisfies the definition $M^*$ and $\otimes$, as detailed in Section 5, cannot satisfy Weyuker Property 9.

Generalization 2

Definition 4. Let $G = \{G_1, G_2, \ldots\}$ be a collection of digraphs and let $\otimes_G$ be the operation of concatenation in $G$ such that $G_i \otimes_G G_j \in G$, $|V_{G_i}| \leq |V_{G_i}| + |V_{G_i}|$, and $|E_{G_i}| \leq |E_{G_i}| + |E_{G_i}|$.

Theorem 3. Let $M^*: G \to R$ be a structural metric, where $G = \{G_1, G_2, \ldots\}$ and $R$ is the set of real numbers. If $M^*(G_1) = |V_{G_1}|$ or else if $M^*(G_1) = |E_{G_1}|$ or else if $M^*(G_1)$ is a linear combination of $|V_{G_1}|$ and $|E_{G_1}|$, then $M^*$ cannot satisfy Property 9 under $\otimes_G$. 
Comments on “On the Applicability of Weyuker Property 9 to Object-Oriented Structural Inheritance Complexity Metrics”

Lu Zhang and Dan Xie
IEEE Trans on Soft Eng May 02

Def 3.1 – contradict cor 1

Definition 3.1. Consider the digraph $G = (V, E)$ as an abstraction of inheritance. Let $v$ be any vertex in $V$. The metric $M^*(v)$ is defined as the number of vertices to which there are at least two paths from $v$ in $G$.

Fig 1 – exception to corollary 1

Argument against corollary 1

As depicted in Fig. 1, $G = (V, E)$ is a digraph and $G' = (V', E')$ is the digraph obtained by concatenation of $u$ and $w$, where $u, w \in V$. In $G$, there are two paths from $u$ to $r$ and two paths from $w$ to $r$; and, for any vertex other than $r$, there is at most one path from $u$ to it, and, at most, one path from $w$ to it. In $G'$, there are two paths from $u \oplus w$ to $r$, two paths from $u \oplus w$ to $h$, and four paths from $u \oplus w$ to $r$ and, for any vertex other than $u, h, b, r$, there is, at most, one path from $u \oplus w$ to it. Based on Definition 3.1, $M^*(u) = M^*(w) = 1$, but $M^*(u \oplus w) = 3$. As the definition of $M^*$ satisfies the conditions required in “Corollary 1,” the conclusion in “Corollary 1” cannot be drawn.

Longest path

Definition 3.2. Consider the digraph $G = (V, E)$ as an abstraction of inheritance. Let $v$ be any vertex in $V$. The metric $M^{**}(v)$ is defined as the maximum length of the longest path from $v$ to a vertex such that there are at least two paths from $v$ to it in $G$.

Def 3.2 – contradiction of th 2

As depicted in Fig. 2, $G = (V, E)$ is a digraph and $G' = (V', E')$ is the digraph obtained by concatenation of $u$ and $w$, where $u, w \in V$. In $G$, there are two paths from $u$ to $r$ and, for any vertex other than $r$, there is, at most, one path from $u$ to it. There are two paths from $w$ to $h$ and, for any vertex other than $h$, there is, at most, one path from $w$ to it. By Definition 3.2, $M^{**}(u) = M^{**}(w) = 2$. In $G'$, there are two paths from $u \oplus w$ to $r$, two paths from $u \oplus w$ to $h$, and two paths from $u \oplus w$ to $r$ and, for any vertex other than $r, h, b$, and $r$, there is, at most, one path from $u \oplus w$ to it. By Definition 3.2, $M^{**}(u \oplus w) = 5$. As the definition of $M^{**}$ satisfies the conditions required in “Theorem 2,” the conclusion in “Theorem 2” cannot be drawn.
Fig 2 – exception to corollary 2

Zhang conclusion

As Weyuker Property 9 is not defined with inheritance metrics in mind, it is quite natural to find that this property is not applicable to many previously proposed inheritance metrics. However, it cannot be theoretically guaranteed that all the sensible inheritance metrics in the classes defined by Gursaran and Roy will definitely violate the property. We simply suggest ignoring Weyuker Property 9 while evaluating an inheritance metric for object-oriented software.